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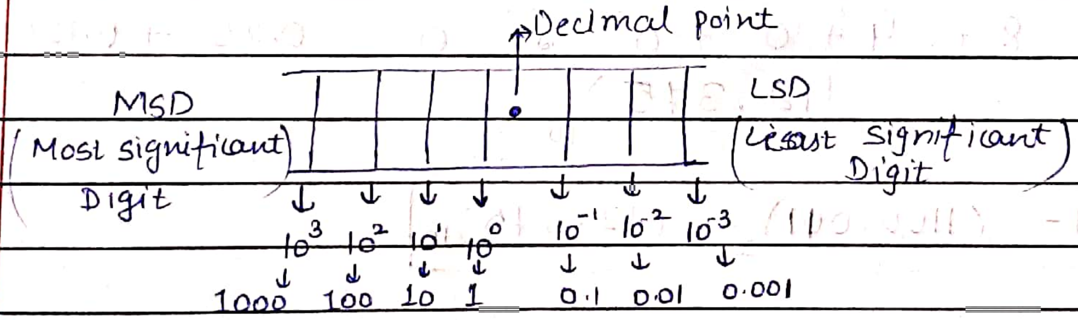
Unit : 1 "Boolean Algebra And Combinational Circuits"

Number System -

It is a set of rules and symbols used to represent numbers. It's knowledge is very essential because the design and organisatio of computer is dependent upon the number system. It is of 4 types -

(1)	Binary	2	Base (b) or Radix (r)
(2)	Octal	8	
(3)	Decimal	10	
(4)	Hexadecimal	16	

- Decimal Number system



Ques Represent decimal number 786.182 in terms of power of 10.

Ans $(7\ 8\ 6\ .\ 1\ 8\ 2)_{10}$

$7\times10^2 \quad 8\times10^1 \quad 6\times10^0 \quad + \quad 1\times10^{-1} \quad 8\times10^{-2} \quad 2\times10^{-3}$

$700 + 80 + 6 + 0.1 + 0.08 + 0.002$

$\boxed{(786.182)_{10}}$ Hence solved. - -

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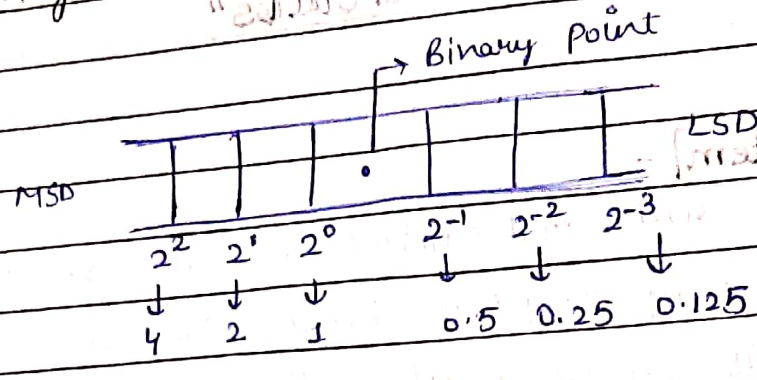
- Binary Number System (0, 1)



Ques Represent the given number in terms of power of 10 and find its decimal equivalent.

$$N = (1100.011)_2$$

Ans

$$(1\ 1\ 0\ 0\ .\ 0\ 1\ 1)_2$$

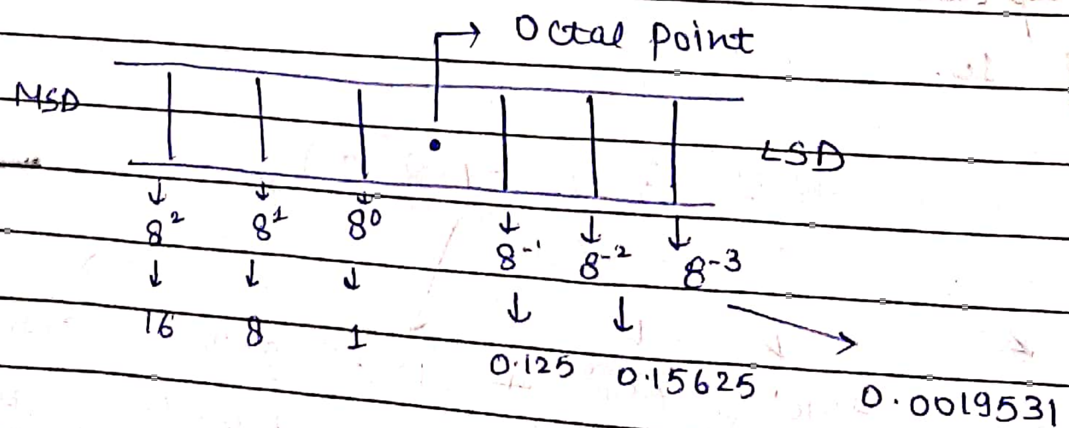
$$1\times2^3 \quad 1\times2^2 \quad 0\times2^1 \quad 0\times2^0 + 0\times2^{-1} \quad 1\times2^{-2} \quad 1\times2^{-3}$$

$$8 + 4 + 0 + 0 + 0 + 0.25 + 0.125$$

$$(12.375)_{10}$$

$$\boxed{N = (1100.011)_2 = (12.375)_{10}}$$

- Octal Number System (0, 1, 2, 3, 4, 5, 6, 7)



Ques Represent the given number in terms of power of 8 and find its decimal equivalent.

$N = (431.32)_8$

Ans $(4\ 3\ 1\ .\ 3\ 2)_8$

$4\times 8^2 \quad 3\times 8^1 \quad 1\times 8^0 \ + \ 3\times 8^{-1} \quad 2\times 8^{-2}$

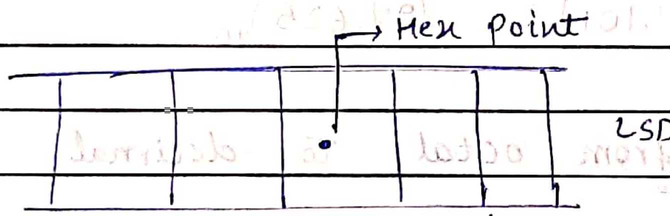
$256 + 24 + 1 + 3\times 0.125 + 2\times 0.15625$

$(281.40625)_{10}$

$$\boxed{N = (281.40625)_{10} = (431.32)_8}$$

- Hexadecimal Number System (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15



$16^2 \quad 16^1 \quad 16^0 \quad 16^{-1} \quad 16^{-2} \quad 16^{-3}$

$256 \quad 16 \quad 1 \quad \frac{1}{16} \quad \frac{1}{256} \quad \frac{1}{4096}$

Ques Represent hexadecimal number 3FB in power of 16 and find its equivalent decimal number.

Ans $N = (3FB)_{16}$ $\quad (\because F=15,\ B=11)$

$3\times 16^2 \quad 15\times 16^1 \quad 11\times 16^0$

$768 + 240 + 11 = (1019)_{10}$

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$$N = (3FB)_8 = (1019)_{10}$$

Number Base Conversion -

- Conversion from binary to decimal

$(11011.101)_2 \rightarrow (?)_{10}$

$(1 \quad 1 \quad 0 \quad 1 \quad 1 \,.\, 1 \quad 0 \quad 1)_2$

$1\times2^4 \quad 1\times2^3 \quad 0\times2^2 \quad 1\times2^1 \quad 1\times2^0 + 1\times2^{-1} \quad 0\times2^{-2} \quad 1\times2^{-3}$

$16 + 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125$

$(27.625)_{10}$

$$(11011.101)_2 = (27.625)_{10}$$

- Conversion from octal to decimal

$(521.63)_8 \Rightarrow (?)_{10}$

$(5 \quad 2 \quad 1 \,.\, 6 \quad 3)_8$

$5\times8^2 \quad 2\times8^1 \quad 1\times8^0 + 6\times8^{-1} \quad 3\times8^{-2}$

$320 + 16 + 1 + 0.75 + 0.0468$

$(337.7968)_{10}$

$$(521.63)_8 = (337.7968)_{10}$$

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- Conversion from hexadecimal to decimal :

$(2B.48)_{16} \rightarrow (?)_{10}$

$$(2 \quad B \quad . \quad 4 \quad 8)_{16}$$

$$2\times16^1 \quad 11\times16^0 + 4\times16^{-1} \quad 8\times16^{-2}$$

$$32 + 11 + 0.25 + 0.03125$$

$$43 + 0.28125$$

$$(43.28125)_{10}$$

$(2B.48)_{16} = (43.28125)_{10}$

- Conversion from decimal to binary

$(37.65625)_{10} \rightarrow (?)_2$

2	37	
2	18	– 1
2	9	– 0
2	4	– 1
2	2	– 0
	1	– 0

~~10100~~ $(100101)_2$

0.65625 × 2 = 1.31250 → 1

0.31250 × 2 = 0.62500 → 0

0.625000 × 2 = 1.250000 → 1

0.250000 × 2 = 0.500000 → 0

0.500000 × 2 = 1.000000 → 1

$(10101)_2$

$(37.65625)_{10} = (100101.10101)_2$

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- Conversion from decimal to octal

$(34.45)_{10} \rightarrow (?)_8$

8	34
8	4 − 2
	0 − 4

$(42)_8$

$$\begin{array}{r} 0.45 \\ \times 8 \\ \hline 3.60 \end{array} \downarrow 3 \qquad \begin{array}{r} 0.60 \\ \times 8 \\ \hline 4.80 \end{array} \downarrow 4 \qquad \begin{array}{r} 0.80 \\ \times 8 \\ \hline 6.40 \end{array} \downarrow 6 \qquad \begin{array}{r} 0.40 \\ \times 8 \\ \hline 3.20 \end{array} \downarrow 3$$

$$\begin{array}{r} 0.20 \\ \times 8 \\ \hline 1.60 \end{array} \downarrow 1$$

$(0.34631)_8$

$(34.45)_{10} = (42.34631)_8$

- Conversion from decimal to hexadecimal

$(7825.760)_{10} \rightarrow (?)_{16}$

16	7825
16	489 − 1
16	30 − 9
	1 − 14

$(1E91)_{16}$

$(\because 14 = E)$

$$\begin{array}{r} 0.760 \\ 16 \\ \hline 12.16 \end{array} \downarrow 12 \qquad \begin{array}{r} 0.16 \\ 16 \\ \hline 2.56 \end{array} \downarrow 2$$

$$\begin{array}{r} 0.56 \\ 16 \\ \hline 8.96 \end{array} \downarrow 8 \qquad \begin{array}{r} 0.96 \\ 16 \\ \hline 15.36 \end{array} \downarrow 15$$

$(C28F)_{16}$

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$(7825.760)_{10} = (1E91.C28F)_{16}$

- Conversion from binary to octal

(1.) $(10110)_2 = (?)_8$

10110

$\underbrace{010}_{2}\ \underbrace{110}_{6}$

$(10110)_2 = (26)_8$

(2.) $(11100110.101111101)_2 = (?)_8$

$\underbrace{011}_{3}\ \underbrace{100}_{4}\ \underbrace{110}_{6} . \underbrace{101}_{5}\ \underbrace{111}_{7}\ \underbrace{101}_{5}$

$(11100110.101111101)_2 = (346.575)_8$

- Conversion from binary to Hexadecimal

$(10110110.101111001)_2 \rightarrow (?)_{16}$

$\underbrace{1011}_{11}\ \underbrace{0110}_{6} . \underbrace{1011}_{11}\ \underbrace{1100}_{12}\ \underbrace{1000}_{8}$

$(B6.BC8)_{16}$

$(10110110.101111001)_2 = (B6.BC8)_{16}$

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- Conversion from Octal to Hexadecimal

$(536.21)_8 = (?)_{16}$

$(5 \quad 3 \quad 6 \;.\; 2 \quad 1)_8$

↓ ↓ ↓ ↓ ↓

101 011 110 . 010 001

$(101011110.010001)_2$ (octal to binary)

0001 0101 1110 . 0100 0100

1 5 E . 4 4

$(536.21)_8 = (15E.44)_{16}$

- Conversion from hexadecimal to octal

$(3CFB.2E)_{16} = (?)_8$

$(3 \quad C \quad F \quad B \;.\; 2 \quad E)_{16}$

$(3 \quad 12 \quad 15 \quad 11 \;.\; 2 \quad 14)_{16}$

↓ ↓ ↓ ↓ ↓ ↓

$(0011 \quad 1100 \quad 1111 \quad 1011 \;.\; 0010 \quad 1110)_2$ (hexadecimal to binary)

$(000 \quad 011 \quad 110 \quad 011 \quad 111 \quad 011 \;.\; 001 \quad 011 \quad 100)_2$

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

$(0 \quad 3 \quad 6 \quad 3 \quad 7 \quad 3 \;.\; 1 \quad 3 \quad 4)_8$

$(3CFB.2E)_{16} = (36373.134)_8$

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Ques Calculate the value of base X if:

(i) $(225)_X = (341)_8$

(ii) $(100)_X = (61)_8$

Ans (i) $(341)_8$

$3\times8^2 + 4\times8^1 + 1\times8^0$

$192 + 32 + 1$

$(225)_{10}$

A/c to question,

$(225)_{10} = (225)_X$

$\boxed{X = 10}$

(ii) $(100)_X = (61)_8$

$(61)_8 \qquad (100)_X$

$6\times8^1 + 1\times8^0 \qquad = 1\times X^2 + 0\times X^1 + 0\times X^0$

$48 + 1 = 49 \qquad = X^2$

$X = \sqrt{49}$

$\boxed{X = 7}$

$\boxed{\text{Complement}}$ –

Complement of number

1's Complement | 2's complement

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(1.) 1's Complement of $(10010)_2$

$(10010)_2$
↓↓↓↓↓
$(01101)_2$

(2.) 2's Complement of $(1010)_2$

$(10\ 10)_2$
↓↓ ↓↓
$(01\ 01)_2$ ← 1's complement
+1
$(01\ 1\ 0)_2$ ← 2's complement

Unsigned and Signed Binary Numbers —

- The numbers without any positive and negetive sign are known as unsigned binary numbers.

- The numbers which are represented by the sign magnitude format are known as signed binary numbers.

Negative (−)

(−) singn tends to 1

example −6

(−) 6

1 110 = 1110

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Positive (+)

(+) sign tends to 0

example : + 6

(+) 6

0 110 = 0110

Decimal Number	Signed Magnitude form
+7	0111
+6	0110
+5	0101
+4	0100
+3	0011
+2	0010
+1	0001
+0	0000
−0	1000
−1	1001
−2	1010
−3	1011
−4	1100
−5	1101
−6	1110
−7	1111

Signed magnitude form is not possible for 4-bit representation i.e, 8 and more than 8 decimal number.

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Logic Gates –

It is an electronic circuit which makes the logical decisions. They have one or more inputs and only one output. Most common logic gates used are AND, OR, NOT, NAND, NOR, XOR, XNOR gates.

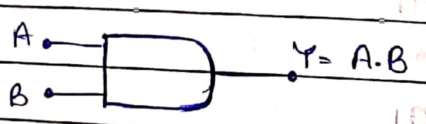
LOGIC GATES

Basic Gates	Universal Gates	Special Purpose Gates
↳ AND	↳ NAND	↳ XOR
↳ OR	↳ NOR	↳ XNOR
↳ NOT		

• AND Gate

It gives logical multiplication output. It has two or more inputs and single output.

Mathematically expression, $Y = A \cdot B$



Inputs		Outputs
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

The output of AND Gate is low even if one input is low.

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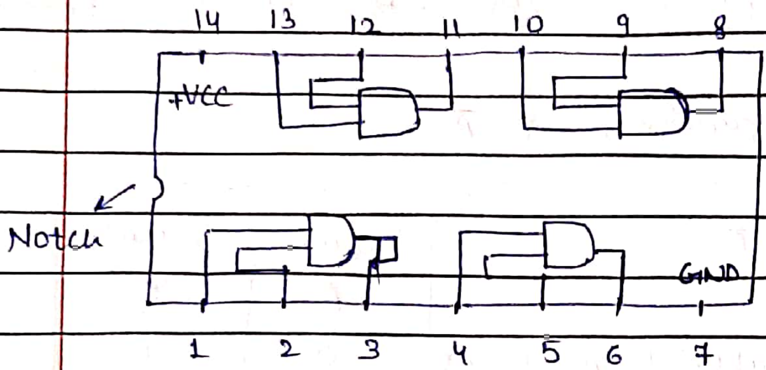
Pin diagram of two input AND gate

~~IC 4708~~

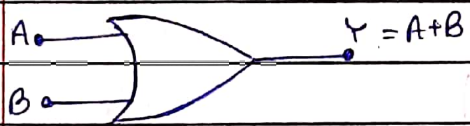
IC 7408



- OR Gate

It performs logical addition. It has two or more inputs and single output.

Mathematically expression, $Y = A+B$

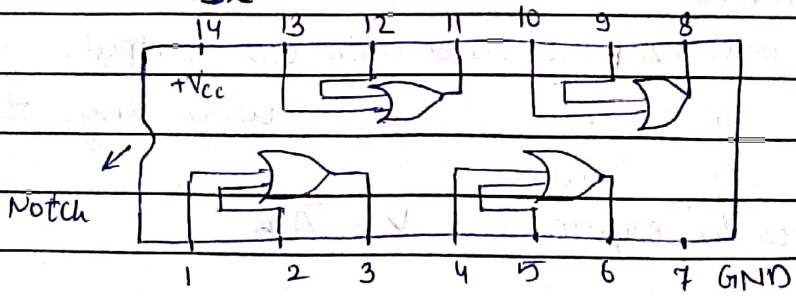


Inputs		Outputs
A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

Output of OR gate is high only when any one of its input is high.

Pin diagram of two inputs OR gate.

IC 7432



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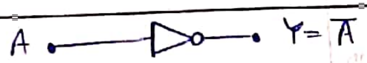
- **NOT Gate**

It performs the basic logical function called complementation. NOT gate is also called inverter. It has one input and one output. It converts one logic ~~gate~~ level into opposite logic level.

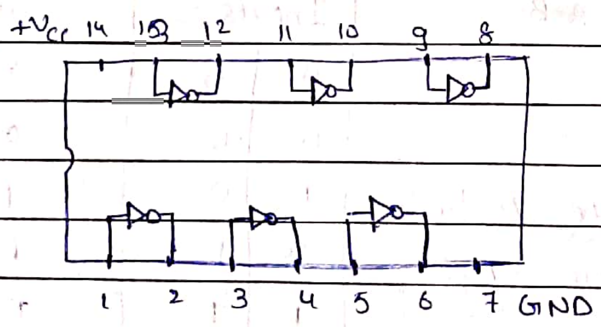
Mathematically expression, $Y = \overline{A}$



Inputs	Outputs
A	Y
0	1
1	0

when a high is applied as a input a low appears on its output and when a low is applied as a input a high appears on its output.

Pin diagram of NOT gate
IC 7404



- **NAND Gate**

It is a combination of AND gate and NOT gate. It has two or more inputs and only one output. The output of NAND gate is high when any of the input is low.

Mathematically expression, $Y = \overline{A.B}$

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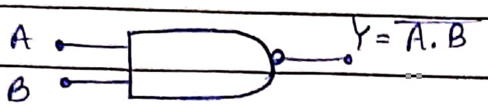
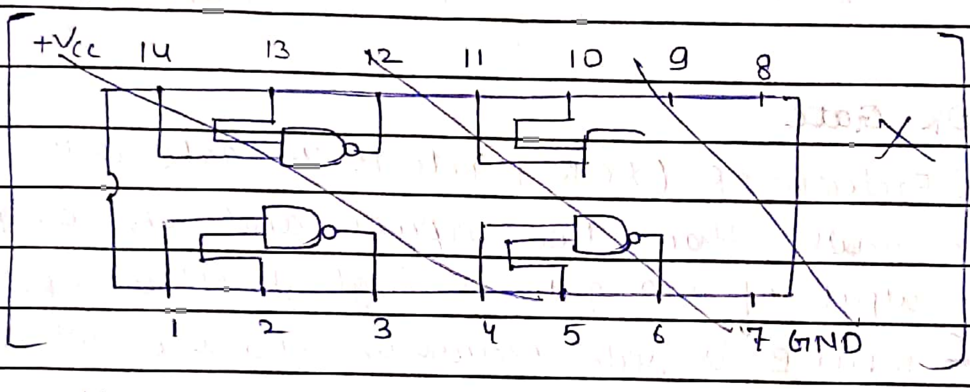
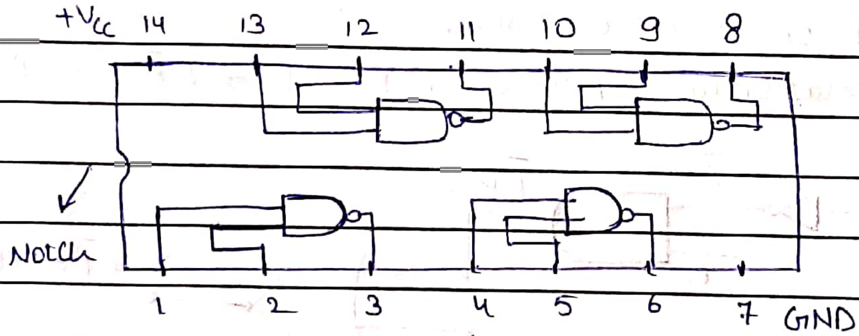


Inputs		Outputs
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Pin diagram of NAND Gate

IC 7400

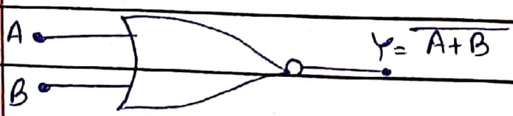




- **NOR Gate**

It is a combination of OR gate and NOT gate. It has two or more inputs and only one output. The output of NOR gate is high when both inputs are low.

Mathematically expression, $Y = \overline{A+B}$



Inputs		outputs
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

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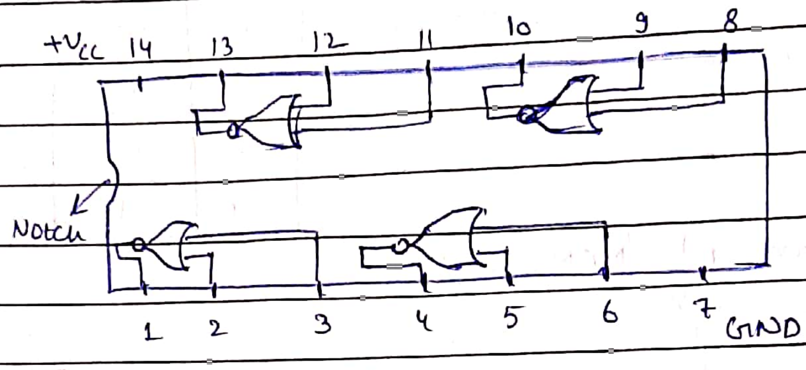
Pin diagram of NOR Gate

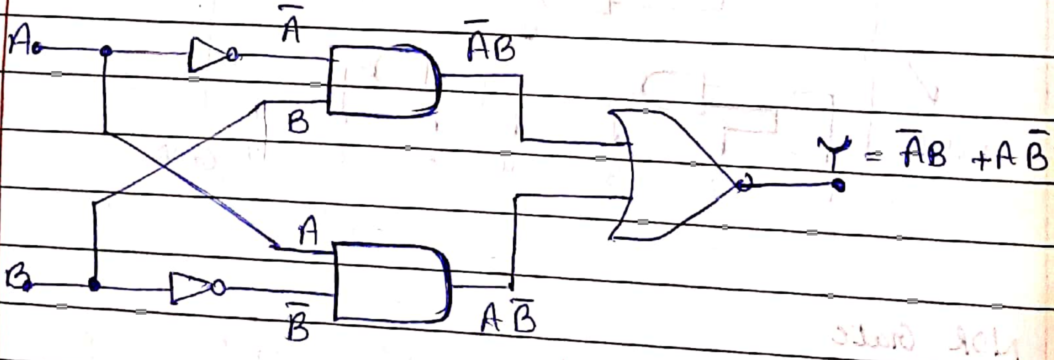
IC 7402



• XOR Gate

An Enclusive OR (XOR) gate is the gate with two or more than two inputs and one output. The output of XOR gate is high if either input A ~~and~~ or input B is high exclusively and low when both are '1' or '0' at a time.

Mathematically expression, $Y = \bar{A}B + A\bar{B} = A \oplus B$



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Inputs		Outputs
A	B	Y
0	0	0
0	1	1
0	1	1
1	1	0

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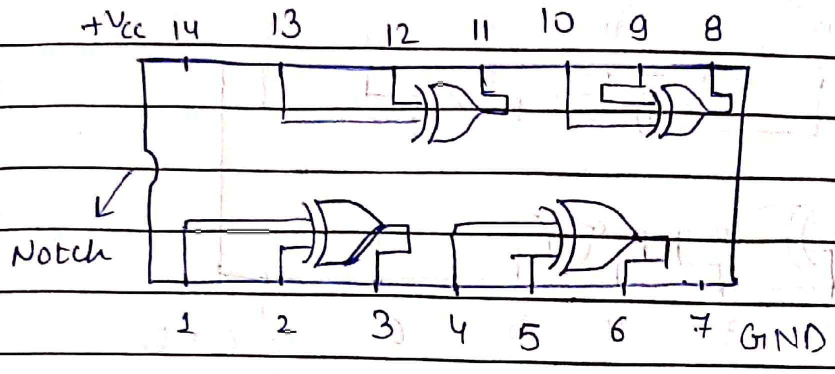
Pin diagram of XOR gate
IC 7486



• XNOR Gate

An exclusive NOR (XNOR) gate is the gate with two or more than two inputs and one output. The output of XNOR gate is high if both input A and input B are high exclusively or same as input A and input B are low exclusively and low when both are different.

Mathematically expression, $Y = A \odot B = AB + \bar{A}\bar{B}$

Inputs		Outputs
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$$\overline{A \oplus B} = \overline{A\bar{B} + \bar{A}B}$$
$$= (\overline{A\bar{B}})(\overline{\bar{A}B})$$
$$= (\bar{A}\,\bar{\bar{B}})(\bar{\bar{A}}\,B)$$
$$= (\bar{A}+B)(A+\bar{B})$$
$$= \bar{A}A + \bar{A}\bar{B} + AB + \bar{B}B$$
$$= 0 + \bar{A}\bar{B} + AB + 0$$
$$= \bar{A}\bar{B} + AB$$
$$= A \odot B$$

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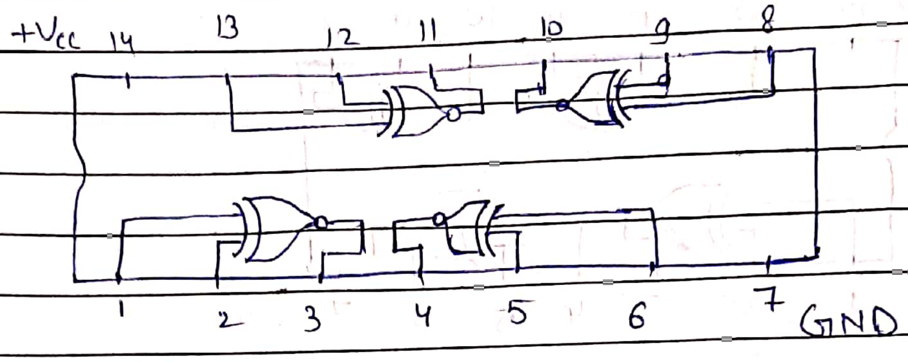
Pin diagram of XNOR gate

IC 74266



Theorems of Boolean Algebra –

AND laws

$A.0 = 0$ $\quad$ $A+0 = A$

$A.1 = A$ $\quad$ $A+1 = 1$

$A.A = A$ $\quad$ $A+A = A$

$A.\bar{A} = 0$ $\quad$ $A+\bar{A} = 1$

OR laws

Commutative law: $A.B = B.A$ $\quad$ $A+B = B+A$

Associative law: $A.(B.C) = (A.B).C$ $\quad$ $A+(B+C) = (A+B)+C$

Distributive laws: $A+(B.C) = (A+B)(A+C)$ $\quad$ $A(B+C) = AB+AC$

Absorption law: $A+(A.B) = A$ $\quad$ $A(A+B) = A$

Double inversion law: $\bar{\bar{A}} = A$

→ Consensus theorem: $AB+\bar{A}C+BC = AB+\bar{A}C$

→ De-Morgan's theorem: $\overline{A+B} = \bar{A}.\bar{B}$ $\quad$ $\overline{A.B} = \bar{A}+\bar{B}$

De-Morgan's theorem –

(a) $\overline{A+B} = \bar{A}.\bar{B}$

It states that $(\bar{A}\cdot\bar{B})$ is the complement of $A+B$

$\therefore$ We have to prove that $(A+B) + \bar{A}\cdot\bar{B} = 1$

$$(A+B) + \bar{A}\cdot\bar{B}$$

$[(A+B)+\bar{A}]\,[(A+B)+\bar{B}]$ by distributive law

$[(A+\bar{A})+B]\,[(B+\bar{B})+A]$ by Associative law

$$[1+B]\;[1+A]$$

$$[1]\,[1]$$

$$1$$

$\therefore$ $(A+B) + \bar{A}\cdot\bar{B} = 1$

(b) $\overline{A\cdot B} = \bar{A} + \bar{B}$

It sates that $(\bar{A}+\bar{B})$ is the complement of $A\cdot B$

$\therefore$ We have to prove the $(A+B)\cdot(\bar{A}\cdot\bar{B}) = 0$

$$(A+B)\,(\bar{A}\cdot\bar{B})$$

$$A(\bar{A}\cdot\bar{B}) + B(\bar{A}\cdot\bar{B})$$

$$A\cdot\bar{A}\cdot\bar{B}A + B\bar{A}\cdot\bar{B}B$$

$$0\cdot BA + B\bar{A}\cdot 0$$

$$0+0$$

$$0$$

$\therefore$ $(A+B)\cdot(\bar{A}\cdot\bar{B}) = 0$

Hence, by using (1) and (2) we concluded that $(\bar{A}\cdot\bar{B})$ is the complement of $(A+B)$

Thus, $(\overline{A+B}) = (\bar{A}\cdot\bar{B})$ and $(\overline{A.B}) = (\bar{A}+\bar{B})$

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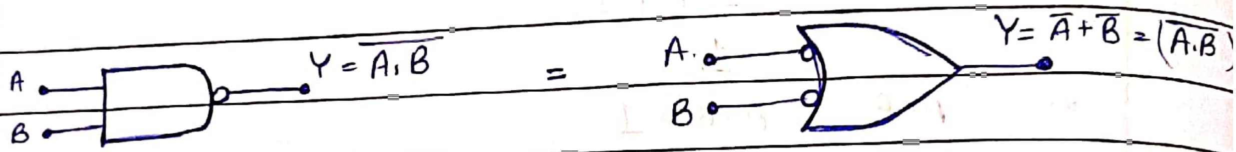
Universal Gates -

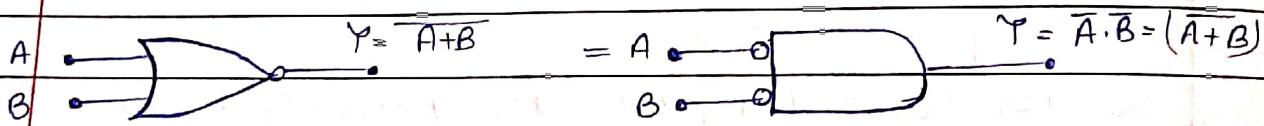
These are those gates which can be used for implementing any gate like AND, OR, XOR, XNOR, NOT or any combination of basic gates.

NAND Gate



NAND gate is also known as bubbled OR gate

NOR Gate

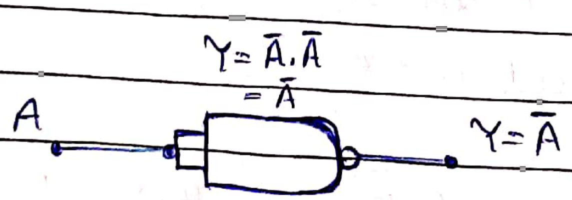


NOR gate is also known as bubbled AND gate

- Universal property of NAND gate

NAND is called universal gate because it can be realize any gate.

→ Realize NOT gate using NAND gate



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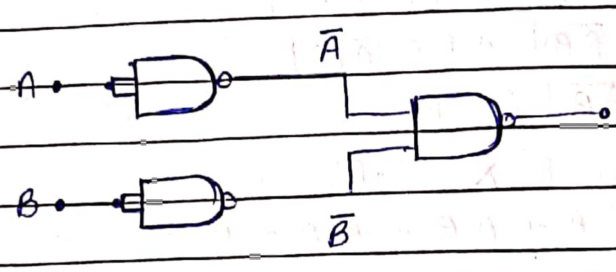
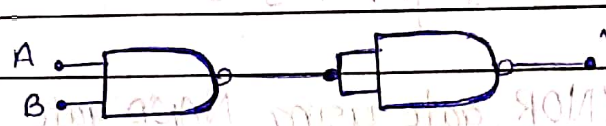
→ Realize OR gate using NAND gates



$Y = \overline{\bar{A}\bar{B}} = \bar{\bar{A}} + \bar{\bar{B}}$

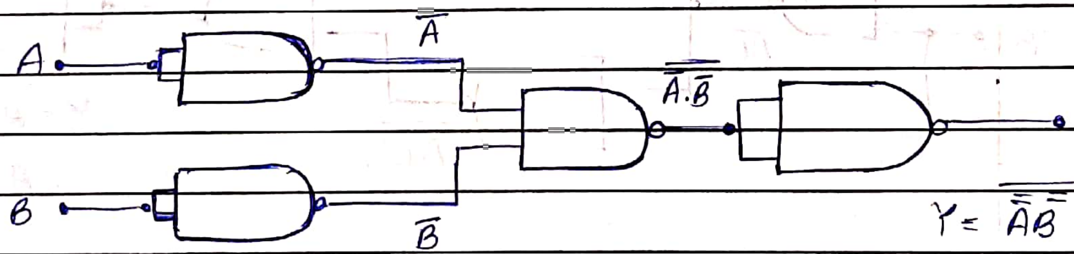
$Y = A + B$

→ Realize AND gate using NAND gates



$Y = \overline{\overline{AB}} = A.B$

→ Realize NOR gate using NAND gates

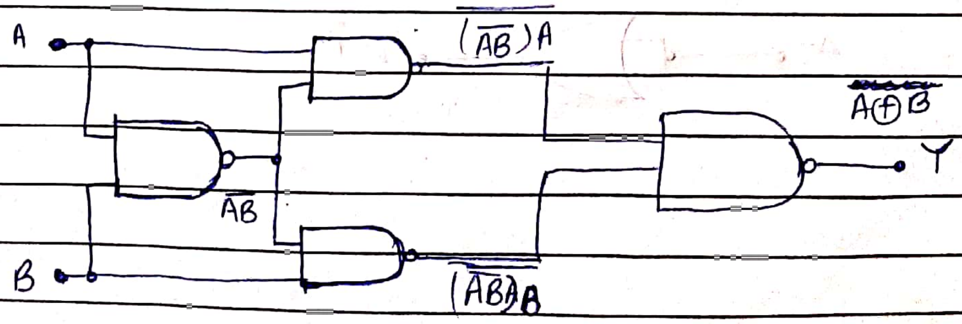


$Y = \overline{\overline{\bar{A}\bar{B}}}$

$= \overline{\bar{\bar{A}} + \bar{\bar{B}}}$

$= \overline{A+B}$

→ Realize a two input EXOR gates using NAND gates



$$Y = \overline{\overline{[A.(\overline{A.B})][B.(\overline{A.B})]}}$$

$$Y = [\overline{\overline{A.(\overline{A.B})}}] + [\overline{\overline{B(\overline{A.B})}}]$$

$$Y = [A.(\overline{A.B})] + [B.(\overline{A.B})]$$

$$Y = [\overline{A.B}](A+B)$$

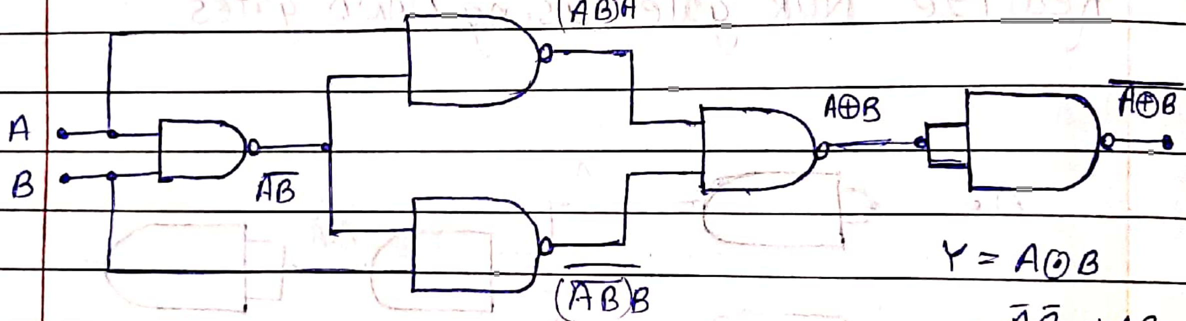
$$Y = (\bar{A}+\bar{B}) \times (A+B)$$

$$Y = \bar{A}.A + \bar{A}.B + \bar{B}.A + \bar{B}.B$$

$$Y = \bar{A}.B + \bar{B}.A = A \oplus B$$

- ~~Universal property of NOR gate~~

→ Realize XNOR gate using NAND gate

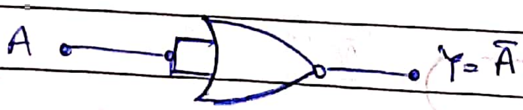


$Y = \overline{A \oplus B}$

$= \bar{A}\bar{B} + AB$

- Universal property of NOR gate

→ Realize NOT gate using NOR gate



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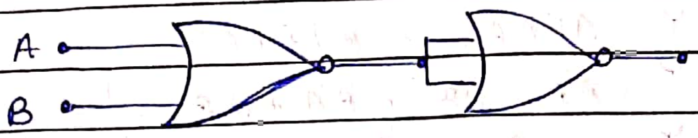
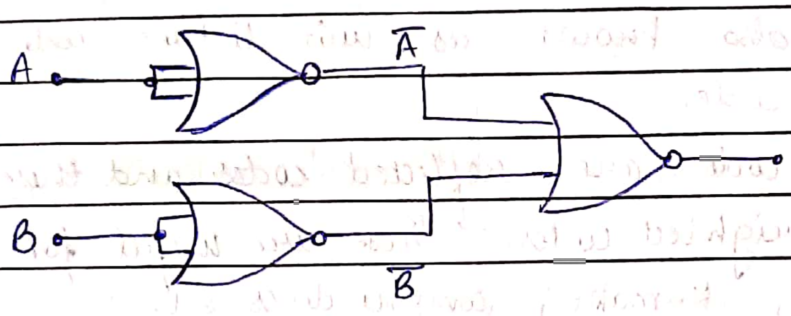
→ Realize ~~AND~~ OR gate using NOR gate



$Y = \overline{\overline{A+B}} = A+B$

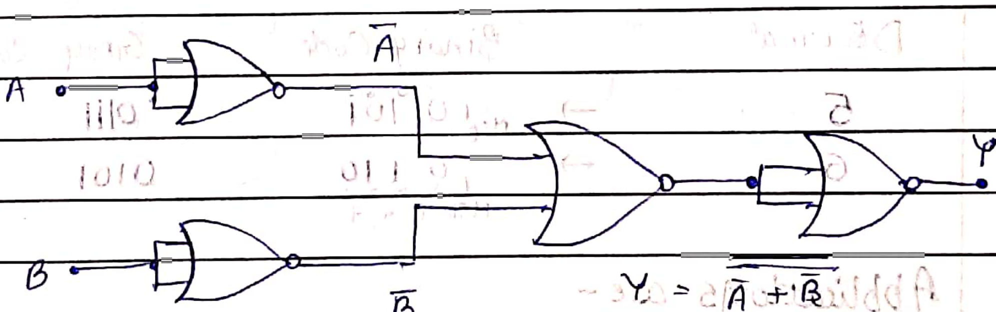
→ Realize AND gate using NOR gate



$Y = \overline{\overline{A} + \overline{B}}$

$= \overline{\overline{A}}\,\overline{\overline{B}} = AB$

→ Realize NAND gate using NOR gate

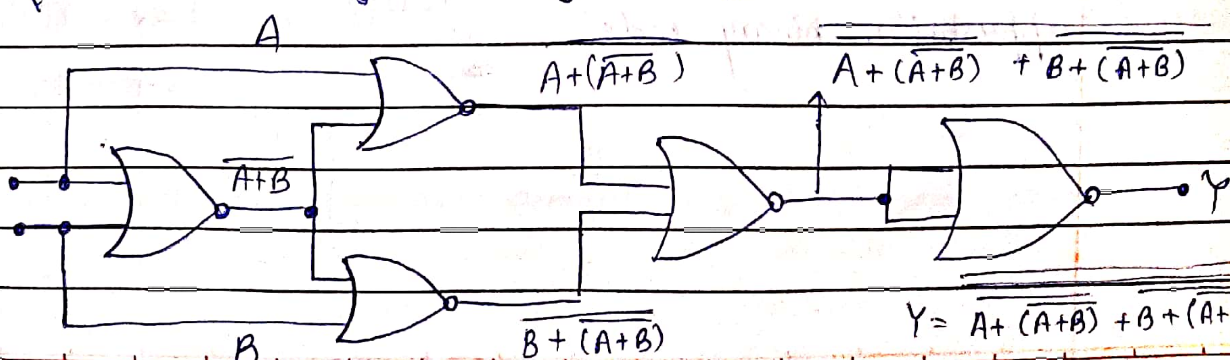


$Y = \overline{A.B}$

$Y = \overline{A} + \overline{B}$

$= \overline{AB}$

→ Realize EXOR gate using NOR gate



$Y = \overline{\overline{A+(\overline{A+B})} + \overline{B+(\overline{A+B})}}$

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$$Y = \overline{A + \overline{(A+B)}} + \overline{B + \overline{(A+B)}}$$

$$Y = \bar{A} \cdot [\overline{\overline{A+B}}] + \bar{B} \cdot [\overline{\overline{A+B}}]$$

$$Y = \bar{A}[A+B] + \bar{B}[A+B]$$

$$Y = \bar{A}A + \bar{A}B + \bar{B}A + \bar{B}B$$

$$Y = \bar{A}B + A\bar{B} = A \oplus B$$

Gray Code -

It is also known as unit distance code and cyclic code.

These codes are reflected codes and these are non-weighted codes. These are useful for input-output devices, K-maps, compact discs ect.

Example -

Decimal		Binary Code	Gray Code
5	→	0 101	0111
6	→	0 110	0101

MSD

Applications are -

→ It is mainly used in shaft position encoders.

→ These are used in the optical discs (CD's) to produce an appropriat binary code.

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Binary to Gray code conversion –

→ The MSB of the binary number remains same in Gray code.

→ Move from MSB to LSB and add each adjacent pair of binary digits, if carry comes neglect or discard the carry and write the sum as next Gray code digit.

Ques Convert $(10110110)_2$ to Gray code.

Ans

1 0 1 1 0 1 1 0

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

1 1 1 0 1 1 0 1

carry is discard   carry is discard

$(10110110)_2 = (11101101)_{\text{Gray code}}$

Gray code to Binary Conversion

→ The MSB of the Gray code remains same in binary number.

→ Add each binary digit generated to the next significant bit of the gray code, if carry comes discard the carry and write the sum as next binary bit.

Ques Convert a given Gray Code to binary

$(11011010)_{\text{Gray code}}$

Ans

MSB 1 1 0 1 1 0 1 0

1 0 0 1 0 0 1 1

↳ carry is discard ↳ carry is discard

$(11011010)_{\text{Gray code}} = (10010011)_2$

Ques Convert a given number to Gray Code.

$(286)_{10}$

Ans

2	286
2	143 - 0
2	71 - 1
2	35 - 1
2	17 - 1
2	8 - 1
2	4 - 0
2	2 - 0
	1 - 0

$(286)_{10} = (100011110)_2$

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$$\begin{array}{cccccccccc} & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ \text{MSB} & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$

↳ carry is discard

$(100011110)_2 = (110010001)_{\text{Gray code}}$

Complement arithmetic -

Ques Do the following

(i) $(25)_{10} - (13)_{10}$ in 2's complement arithmetic

(ii) $(13)_{10} - (25)_{10}$ in 2's complement arithmetic

Ans (i) $(25)_{10} + (-13)_{10}$

2	25
2	12 - 1
2	6 - 0
2	3 - 0
	1 - 1

2	13
2	6 - 1
2	3 - 0
	1 - 1

$(13)_{10} = (1101)_2$

$(25)_{10} = (11001)_2$

→ 2's complement of 13,

1's complement . 10010 — 1 tends to negative sign

$$\begin{array}{r} 10010 \\ +1 \\ \hline 10011 \end{array}$$

2's complement . 10011

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Add it to $(25)_{10}$

$$\begin{array}{r} {}^{1\,1} \\ 11001 \\ 10011 \\ \hline 01100 \end{array}$$

carry ← 1 discard

$(11001)_2 - (01101)_2 = (01100)_2$

$\boxed{(25)_{10} - (13)_2 = (12)_2}$

(ii) $(13)_{10} + (-25)_{10}$

2	13	2	25
2	6 − 1	2	12 − 1
2	3 − 0	2	6 − 0
	1 − 1	2	3 − 0
			1 − 1

$(13)_{10} = (1101)_2$

$(25)_{10} = (11001)_2$

2's complement of $(25)_{10}$

1's complement → 00110
+1

2's complement → 00111

Adde it to $(13)_{10}$

$$\begin{array}{r} {}^{1\,1\,1\,1} \\ 01101 \\ 00111 \\ \hline 10100 \end{array}$$

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Since there is no carry, obtain 2's complement and assign negative sign,

$$(10100)_2$$

$$= 01011$$

$$+1$$

$$-01100$$

$(01101)_2 - (11001)_2 = (-01100)_2$

$\boxed{(13)_{10} - (25)_{10} = (-12)_{10}}$ Ans

Ques Do $-58 - 13$ in 2's complement arithmetic.

Ans

2	58
2	29 − 0
2	14 − 1
2	7 − 0
2	3 − 1
	1 − 1

2	13
2	6 − 1
2	3 − 0
	1 − 1

$(+58)_{10} = (111010)$

for 8-bits

$(+58)_{10} = (00111010)_2$

$(+13)_{10} = (1101)_2$

for 8 bits

$(+13)_{10} = (00001101)_2$

(-13) 1's complement $= 11110010$

$+1$

2's complement $= 11110011$

(-58) 1's complement $= 11000101$

$+1$

2's complement $= 11000110$

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$(-58) + (-13)$

```
  1   11
  11000110
+ 11110011
(1)10111001
```

discard carry

2's complement of $(10111001)_2$

1's compliment → 01000110
+1

2's complement 01000111

sign bit

$(-58)_{10} + (-13)_{10} = -(71)_{10}$

Ques Subtract the following decimal numbers using 9's complement:

(i) $(23)_{10} - (12)_{10}$ (iii) $(236.34)_{10} - (105.93)_{10}$

(ii) $(17)_{10} - (6)_{10}$ (iv) $(105.23)_{10} - (236.34)_{10}$

Ans (i) 23 − 12

The 9's complement of 12 is,

$$\begin{array}{r} 99 \\ -12 \\ \hline 87 \end{array}$$

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$$\begin{array}{r} 23 \\ +\ 87 \\ \hline 1\ 10 \end{array} \leftarrow \text{(Add 9's complement of 12)}$$

$\hookrightarrow +1$ ← (Add end 'around carry)

11 ← final result

$$\boxed{(23)_{10} - (12)_{10} = (11)_{10}}$$

(ii) 17 − 6

The 9's complement of 6 is,

$$\begin{array}{r} 99 \\ -06 \\ \hline 93 \end{array} \leftarrow \text{(9's complement of 6)}$$

$$\begin{array}{r} 17 \\ +\ 93 \\ \hline 1\ 10 \end{array} \leftarrow \text{(Add 9's complement of 6)}$$

$\hookrightarrow +1$

11

$$\boxed{(17)_{10} - (6)_{10} = (11)_{10}}$$

(iii) $(236.34)_{10} - (105.23)_{10}$

$$\begin{array}{r} 999.99 \\ -105.23 \\ \hline 894.76 \end{array} \leftarrow \text{(9's complement of 105.23)}$$

236.34
+894.76

(1) 138.10
+1

138.11 Ans.

(iv) $(105.23)_{10} - (236.34)_{10}$

999.99
−236.34

763.65 ← (9's complement of 236.34)

105.23
+763.65

868.88

There is no carry at MSD, thus the result is −ve.

999.99
−868.88

− 131.11 ← (9' complement of result) and assign −ve sign

Ques Subtract $(18)_{10} - (24)_{10}$ using 9's complement

Ans

99
− 24

75 ← (9's complement of 24)

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$$\begin{array}{r} 18 \\ +75 \\ \hline 93 \end{array}$$

There is no carry at MSD, thus the the result is -ve.

$$\begin{array}{r} 99 \\ -93 \\ \hline -\ 06 \end{array}$$ ← (9's complement of result) and assign -ve sign

$\boxed{(18)_{10} - (24)_{10} = (-6)_{10}}$ Ans

Ques Subtract the following decimal numbers using 10's complement:

(i) $(732.19)_{10} - (213.42)_{10}$

(ii) $(213.42)_{10} - (732.19)_{10}$

Ans (i)

$$\begin{array}{r} 999.99 \\ -213.42 \\ \hline 786.57 \end{array}$$ ← (9's complement of 213.42)

$$\begin{array}{r} 786.57 \\ +1 \\ \hline 786.58 \end{array}$$ ← (10's complement of 213.42)

$$
\begin{array}{r}
732.19 \\
786.58 \\
\hline
(1)\,518.77
\end{array}
$$

← discard carry

~~518.778~~

$$\boxed{(732.19)_{10} - (213.42)_{10} = (518.77)_{10}}$$

(ii)

$$
\begin{array}{r}
999.99 \\
-732.19 \\
\hline
267.80
\end{array}
$$
← (9's complement of 732.19)

$$
\begin{array}{r}
+1 \\
\hline
267.81
\end{array}
$$
← (10's complement of 732.19)

$$
\begin{array}{r}
267.81 \\
+\ 213.42 \\
\hline
481.23
\end{array}
$$

There is no carry so the result is -ve.

$$
\begin{array}{r}
999.99 \\
-481.23 \\
\hline
518.76 \\
+1 \\
\hline
-\ 518.77
\end{array}
$$
← (10's complement and assign -ve sign)

$$\boxed{(213.42)_{10} - (732.19)_{10} = (-518.77)_{10}}$$

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SOP and POS –

Sum of Product form (SOP)

In this case product terms of a function are added.
In this '0' → complemented form = $\bar{A}$
'1' → non-complemented form = A
Sum of product terms are known as 'Minterms' and denoted by 'm'.

Inputs			SOP terms	Minterms Abbreviations
A	B	C		
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	$\bar{A}\bar{B}C$	m_1
0	1	0	$\bar{A}B\bar{C}$	m_2
0	1	1	$\bar{A}BC$	m_3
1	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	$A\bar{B}C$	m_5
1	1	0	$AB\bar{C}$	m_6
1	1	1	ABC	m_7

Product of sum form (POS)

In this case sum terms of a function are multiplied.
In this '0' → non-complemented form = A
'1' → complemented form = $\bar{A}$
Product of sum terms are known as 'Maxterms' and denoted by 'M'.

#

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Inputs			Standard POS terms	Maxterms Abbreviations
A	B	C		
0	0	0	$A + B + C$	M_0
0	0	1	$A + B + \bar{C}$	M_1
0	1	0	$A + \bar{B} + C$	M_2
0	1	1	$A + \bar{B} + \bar{C}$	M_3
1	0	0	$\bar{A} + B + C$	M_4
1	0	1	$\bar{A} + B + \bar{C}$	M_5
1	1	0	$\bar{A} + \bar{B} + C$	M_6
1	1	1	$\bar{A} + \bar{B} + \bar{C}$	M_7

Karnaugh Map –

It is simply a graphical method for representing a boolean function. It is used for the minimization of switching functions but upto 'six' variables.

- Types of K-Maps commonly used -

It consists of

(1) Two variable K-map $2^2 = 4$ squares
(2) Three variable K-map $2^3 = 8$ squares
(3) Four variable K-map $2^4 = 16$ squares
(4) Five variable K-map $2^5 = 32$ squares
(5) Six variable K-map $2^6 = 64$ squares

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Ques Minimize the given function using k-map.
$Y(A,B,C) = \Sigma m(1,3,5,7)$

Ans

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0		1	1	
A 1		1	1	

$\therefore$ $\boxed{Y = C}$

Ques Minimize the given function using k-map.
$Y(A,B,C,D) = \Sigma m(0,2,5,7,8,10,13,15)$

Ans

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1			1
$\bar{A}B$ 01		1	1	
AB 11		1	1	
$A\bar{B}$ 10	1			1

Group2

Group 1

$Y = \text{Group 1} + \text{Group 2}$ ~~+ Group 3~~

$Y = BD + \bar{B}\bar{D}$

Ques Minimize the given function using k-map
$Y(A,B,C,D) = \Pi M(0,1,2,3,6,8,9,10,11,12,13)$

Ans

AB \ CD	CD 00	$C\bar{D}$ 01	$\bar{C}\bar{D}$ 11	$\bar{C}D$ 10
AB 00	0	0	0	0
$A\bar{B}$ 01				0
$\bar{A}\bar{B}$ 11	0	0		
$\bar{A}B$ 10	0	0	0	0

→ Group 1 B

→ Group 2 $A+\bar{C}+D$

Group 4 $C+\bar{A}$

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$Y = (B)(A+\bar{C}+D)(\bar{A}+C)$

Ques Simplify the following logic expression using K-map.

$Y(A,B,C,D,E) = \Sigma m(3,5,6,8,9,12,13,14,19,22,24,25,30)$

Ans

$A=0$ or $\bar{A}$

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	0	1	1 (3)	2
$\bar{B}C$ 01	4	1 (5)	7	1 (6)
BC 11	1 (12)	1 (13)	15	1 (14)
$B\bar{C}$ 10	1 (8)	1 (9)	11	10

$A=1$ or A

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	16	17	1 (19)	18
$\bar{B}C$ 01	20	21	23	1 (22)
BC 11	28	29	31	1 (30)
$B\bar{C}$ 10	1 (24)	1 (25)	27	26

Group 1 = $\bar{B}\bar{C}DE$

Group 2 = $CD\bar{E}$

Group 3 = $B\bar{C}\bar{D}$

Group 4 = $\bar{A}C\bar{D}E$ ($\bar{A}$ include because of left most K-map)

Group 5 = $\bar{A}B\bar{D}$ (")

$Y = \bar{B}\bar{C}DE + CD\bar{E} + B\bar{C}\bar{D} + \bar{A}C\bar{D}E + \bar{A}B\bar{D}$

Ques Minimize the ~~fot~~ given function using K-map.

$Y(A,B,C,D,E) = \Pi M(2,7,8,13,23,24,29)$

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Ans

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	0	1	3	0 2
$\bar{B}C$ 01	4	5	0 7	6
BC 11	12	0 13	15	14
$B\bar{C}$ 10	0 8	9	11	10

$\bar{A}$

1
2
3
4

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	16	17	19	18
$\bar{B}C$ 01	20	21	0 23	22
BC 11	28	0 29	31	30
$B\bar{C}$ 10	0 24	25	27	26

A

Group 1 = $(A + B + C + \bar{D} + E)$

Group 2 = $(B + \bar{C} + \bar{D} + \bar{E}$

Group 3 = $(\bar{B} + \bar{C} + D + \bar{E})$

Group 4 = $(\bar{B} + \bar{C} + D + E)$

$$Y = (A+B+C+\bar{D}+E)(B+\bar{C}+\bar{D}+\bar{E})(\bar{B}+\bar{C}+D+\bar{E})(\bar{B}+\bar{C}+D+E)$$

Que Minimize the given expression using K-map.

$Y(A,B,C,D,E,F) = \Sigma m(4,5,6,7,8,9,20,21,22,23,24,25,34,35,36,37,38,39,50,51,52,53,54,55,56,57)$

Ans

A \ B — $B=0$ — $B=1$

$A=0$, $B=0$:

CD \ EF				
	0	1	3	2
	1 4	1 5	1 7	1 6
	12	13	15	14
	1 8	1 9	11	10

G1

G2

$A=0$, $B=1$:

CD \ EF				
	16	17	19	18
	1 20	1 21	1 23	1 22
	28	29	31	30
	1 24	1 25	27	26

$A=1$, $B=0$:

CD \ EF				
	32	33	1 35	1 34
	1 36	1 37	1 39	1 38
	44	45	47	46
	40	41	43	42

G4

G3

$A=1$, $B=1$:

CD \ EF				
	48	49	1 51	1 50
	1 52	1 53	1 55	1 54
	60	61	63	62
	1 56	1 57	59	58

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$$Y(A,B,C,D,E,F) = G_1 + G_2 + G_3 + G_4$$

Group 1 = $\bar{C}D$
(Common for all 4 variable K-map, thus A and B variables are directly eliminated).

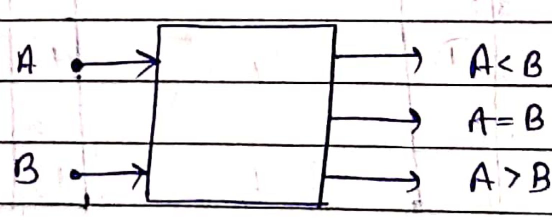
Group 2 = $\bar{A}C\bar{D}\bar{E}$
Group 3 = $BC\bar{D}\bar{E}$
Group 4 = $A\bar{C}\bar{E}$

$$Y(A,B,C,D,E,F) = \bar{C}D + \bar{A}C\bar{D}\bar{E} + BC\bar{D}\bar{E} + A\bar{C}E$$

$\boxed{\text{Magnitude Comparator}}$

Comparator compares two binary numbers.

Example-
Single bit magnitude comparator,



Inputs		Outputs		
A	B	A>B	A=B	A<B
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0