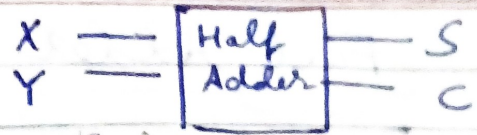


Half Adder

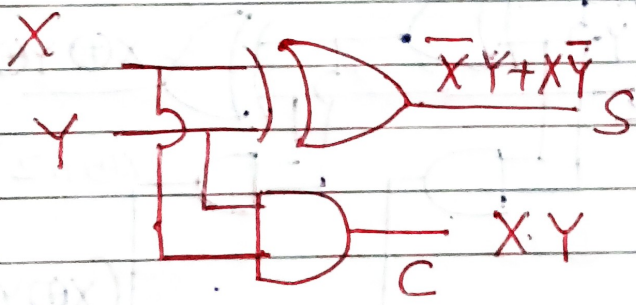
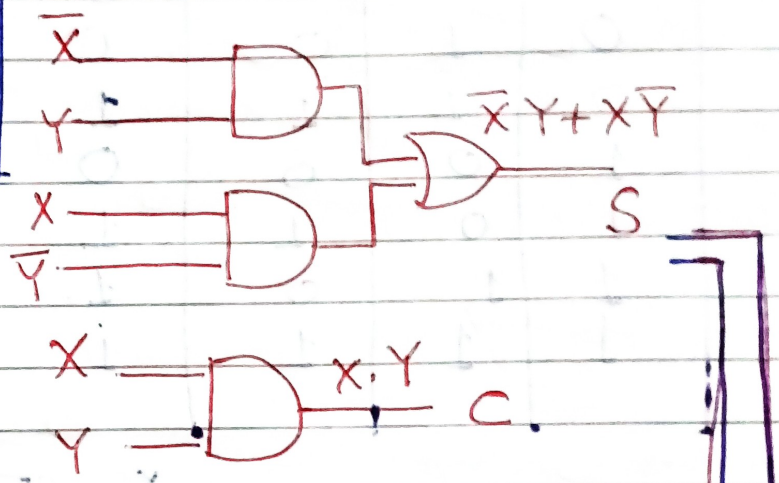


X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Eqn in SOP (m)

$$S = \bar{X}Y + X\bar{Y}$$

$$C = X \cdot Y$$



A	B	S
\bar{A}	\bar{B}	0
\bar{A}	B	1
A	\bar{B}	1
A	B	0

A	B	C
\bar{A}	\bar{B}	0
\bar{A}	B	0
A	\bar{B}	0
A	B	1

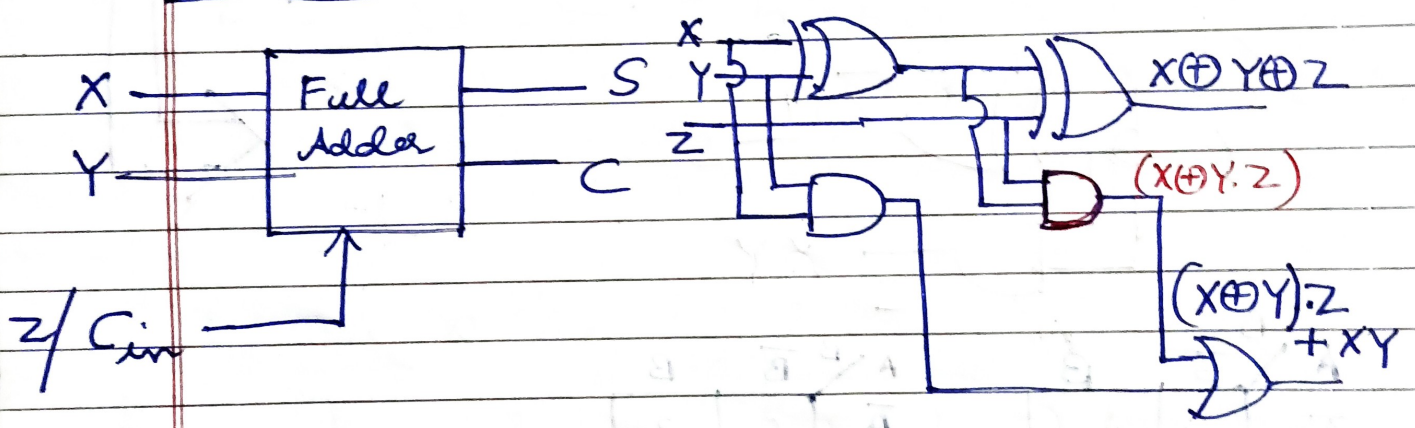
$$S = \bar{A}B + A\bar{B}$$

$$= A \oplus B$$

$$C = A \cdot B$$

Full Adder

X	Y	Z/C _{in}	Carry(C)	Sum(S)
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Eqn. in SOP (min term)

S	$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
\bar{X}	0	1	0	0
X	1	0	1	0
\bar{C}	0	0	1	0
C	0	1	0	1

$$S = X \oplus Y \oplus Z$$

$$C = \bar{X}YZ + X\bar{Y}Z + XYZ + XY\bar{Z}$$

$$= (X \oplus Y) \cdot Z + XY \cdot (Z + \bar{Z})$$

$$\Rightarrow (X \oplus Y) \cdot Z + XY$$

$$\# S = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ$$

or, $\bar{A}\bar{B}\bar{C}_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}C_{in} + ABC$

$$\Rightarrow \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + \bar{B}C)$$

$$\Rightarrow \bar{A}(B \oplus C) + A(\overline{B \oplus C}) \quad \text{--- X}$$

$$\Rightarrow \bar{A} \oplus X$$

$$\Rightarrow \boxed{A \oplus B \oplus C = S}$$

$$\# C = \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

$$= C(\bar{A}B + A\bar{B}) + AB(\bar{C} + C)$$

$$\boxed{\text{Carry} = C \cdot (A+B) + AB = 1}$$

BCD Addition

4-bit
 $2^4 = 16$

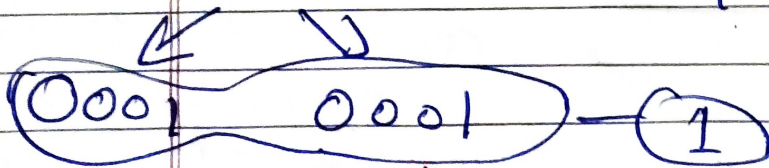
Consider : 0 - 9
Do not consider: 10 - 15
6

Calculation

2	0010	
+ 6	0110	
8	1000	= 8 <i>decimal</i>

3	0011	
+ 8	1000	
11	1011	⇒ Binary

but not BCD



(Out of) more than 9, hence use 6 to add.

	1011	
(6)	0110	
	10000	

or, 00010001 same as 1
4

$$\begin{array}{r} 8 \\ 8 \\ \hline 16 \end{array} \quad \begin{array}{r} 1000 \\ 1000 \\ \hline 10000 \\ \hline 1 \quad 0 \end{array}$$

$$(6) + \begin{array}{r} 10000 \\ 0110 \\ \hline 000 \quad | \quad 0110 \\ \hline 1 \quad 6 \end{array}$$

Answer.

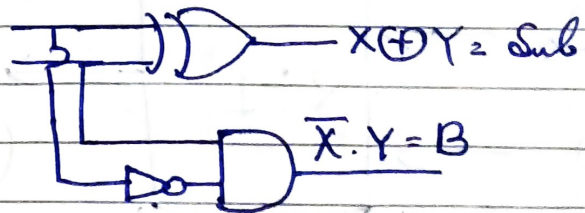
Subtractor

Half Subtractor

X	Y	B	Sub
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

$$X \oplus Y = \text{Sub.}$$

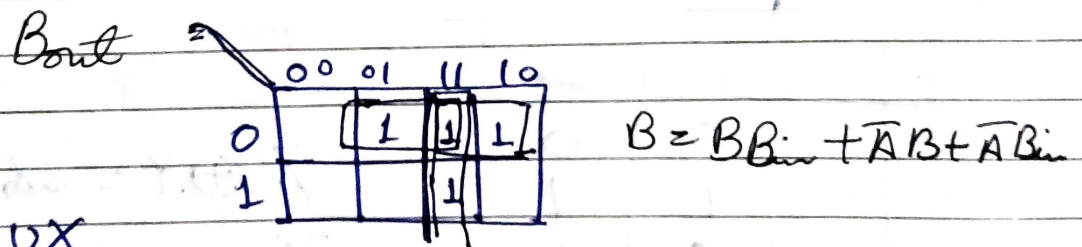
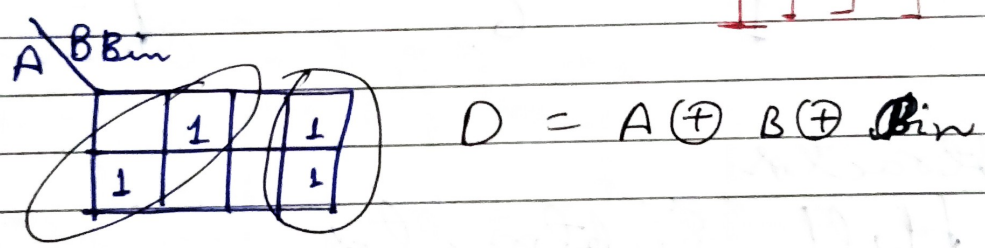
$$\bar{X} \cdot Y = B$$



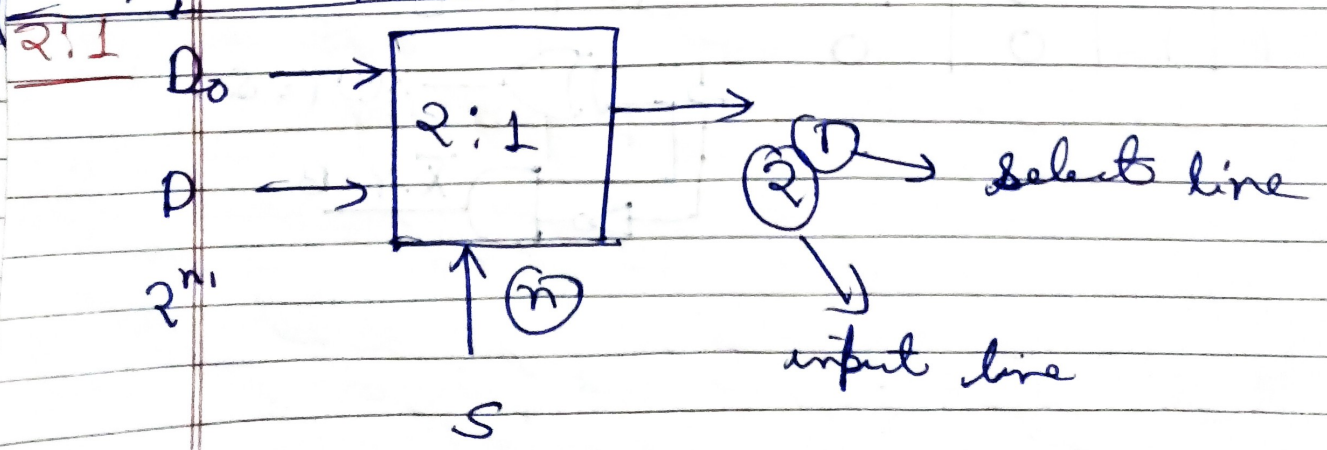
(5)

Full Subtractor

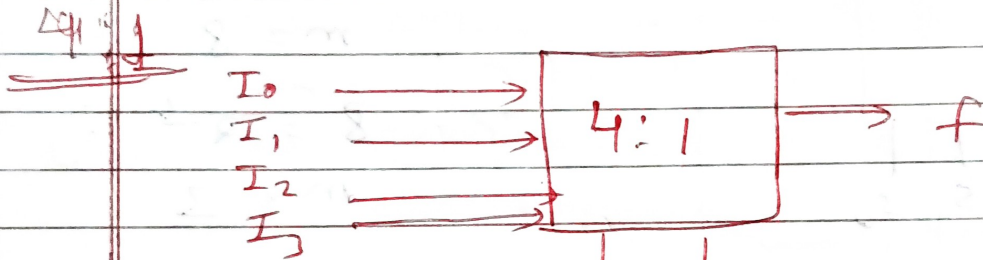
A	B	Bin	D	bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1



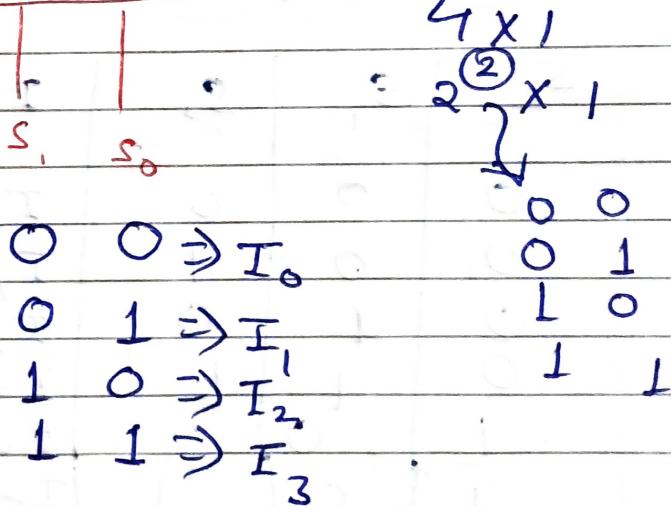
MUX/DEMUX



S	D ₀	D ₁	
0	0	0	0
0	0	1	0
1	1	0	1
1	1	1	1



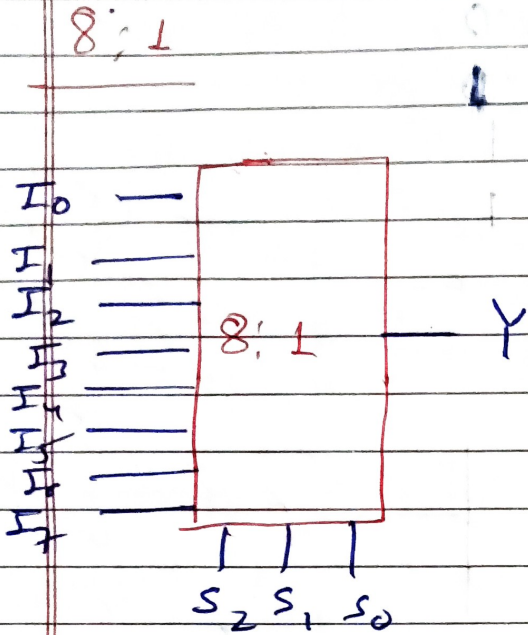
Input ~~bits~~ ⁿ Select lines.



Boolean

$$f = \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3$$

Cogic diagram



$n: 1$
 $8: 1$
 Select lines
 $2^m = n$
 $m = 8$
 $8 = 2^3$
 $m = 3$

S_2	S_1	S_0	I
0	0	0	I_0
0	0	1	I_1
0	1	0	I_2
0	1	1	I_3
1	0	0	I_4
1	0	1	I_5
1	1	0	I_6
1	1	1	I_7

Boolean exp (SOP) :-

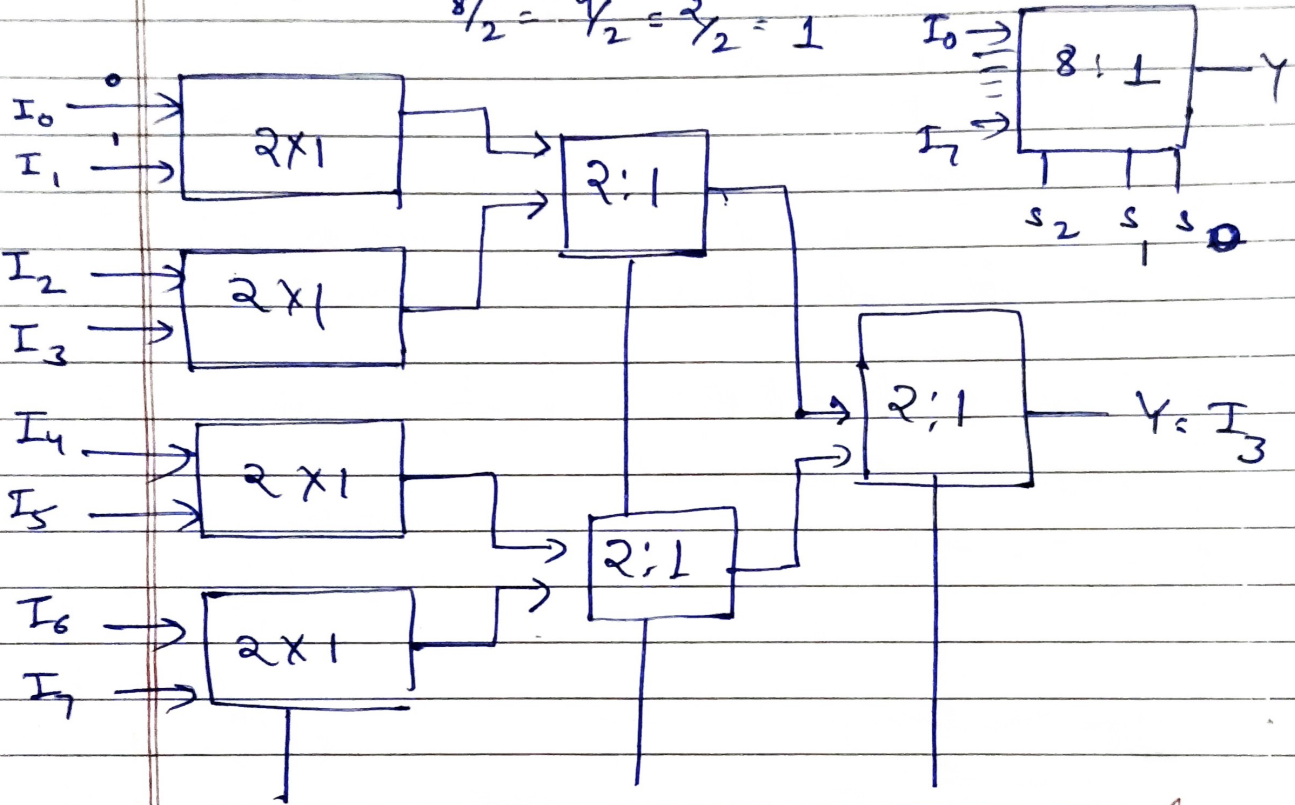
$$\begin{aligned}
 Y = & \overline{S_2} \overline{S_1} \overline{S_0} I_0 + \overline{S_2} \overline{S_1} S_0 I_1 + \overline{S_2} S_1 \overline{S_0} I_2 + \\
 & \overline{S_2} S_1 S_0 I_3 + S_2 \overline{S_1} \overline{S_0} I_4 + S_2 \overline{S_1} S_0 I_5 \\
 & + S_2 S_1 \overline{S_0} I_6 + S_2 S_1 S_0 I_7
 \end{aligned}$$

Logic Diagram

Conversions

Design 8:1 Mux using 2:1 Mux

$$8/2 = 4/2 = 2/2 = 1$$



$s_0 = 0$ $s_1 = 1$ $s_2 = 1$ ~~X~~
 $s_2 = 1$ $s_1 = 1$ $s_0 = 0$

s_0	s_1	s_2	Y
0	0	0	I_0
0	0	1	I_1
0	1	0	I_2
0	1	1	I_3
1	0	0	I_4
1	0	1	I_5
1	1	0	I_6
1	1	1	I_7

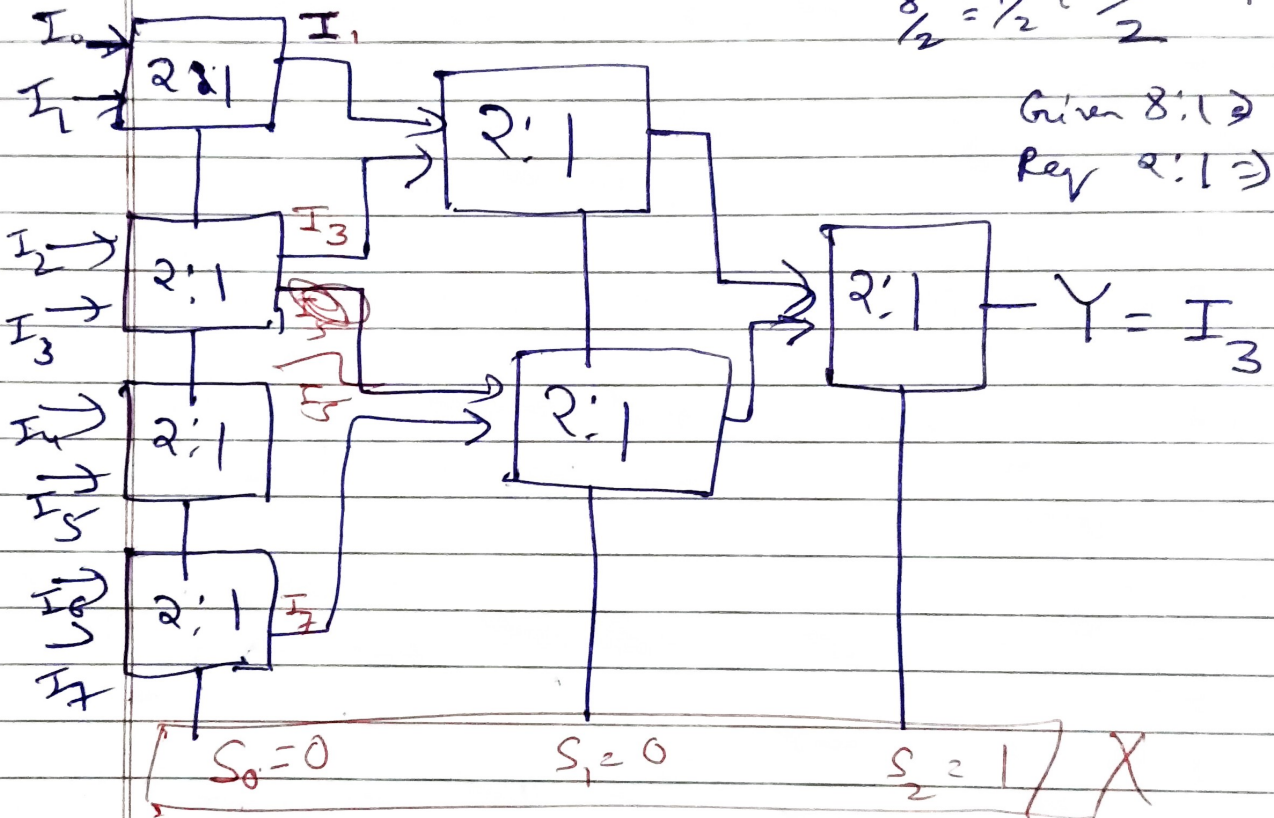
(10)

Design 8:1 using 4:1 & 2:1
or, 16:1 using 4:1 MUX.

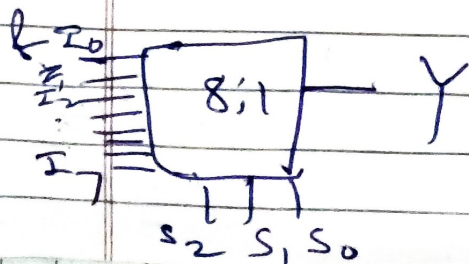
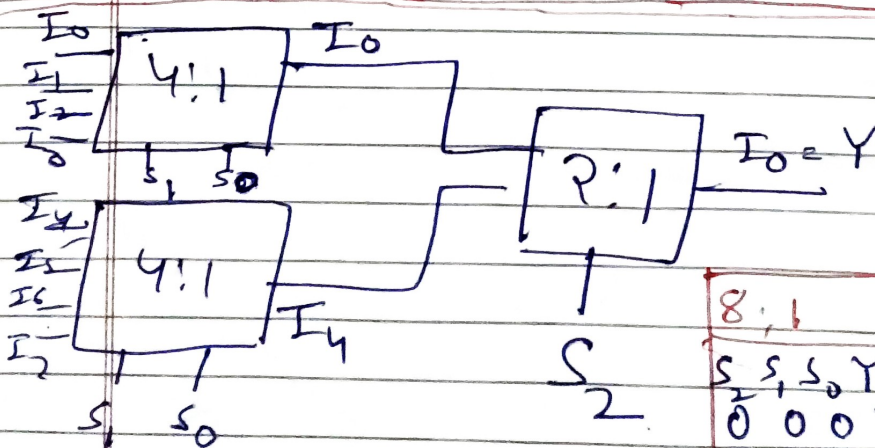
8:1

$$\frac{8}{2} = \frac{4}{2} \cdot \frac{2}{2} = 1 \quad \text{s.l}$$

Given 8:1 \Rightarrow 3
Req 2:1 \Rightarrow 1



$S_2 = 1 \quad S_1 = 1 \quad S_0 = 0$



8:1	4:1	2:1
S_2, S_1, S_0, Y	S_1, S_0, Y	S_0, Y
0 0 0 I_0	0 0 I_0	0 I_0
0 0 1 I_1	0 1 I_1	1 I_1
0 1 0 I_2	1 0 I_2	
...
...

colorz

Date

Page No.

(13)

SOP, Canonical SOP, POS, K-Map.

$$f(x, y, z) = x \cdot \bar{y} \cdot z + x \cdot \bar{y}$$

↳ SOP

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

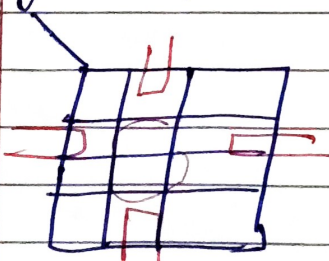
$$f = \underbrace{x \cdot \bar{y} \cdot z}_1 + \underbrace{x \cdot \bar{y} \cdot z}_5 + \underbrace{x \cdot \bar{y} \cdot z}_6 + \underbrace{x \cdot \bar{y} \cdot z}_7$$

$$f = \bar{y}z + \bar{y}z + x\bar{y}z + xyz = \sum m(1, 3, 5, 7)$$

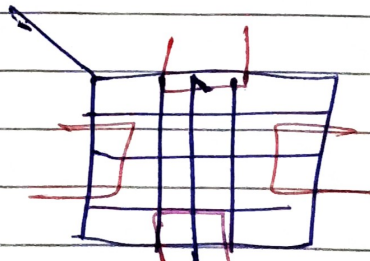
$$\Rightarrow \sum f = m_1 + m_3 + m_5 + m_7$$

Practice of K-map

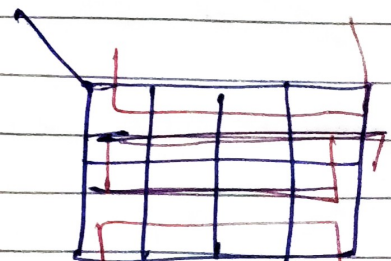
Types of Group



Pair



Quad



Octet

Octet > Quad > Pair

← Priority of grouping.

(15)

P.T.O.

16

Date

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(17)

Q1) $F(A, B) = \sum (0, 2, 3)$

	B	0	1
A	0	0	1
	1	1	0

Red lines indicate prime implicants: P1 (top-left 2x2 square), P2 (bottom-right 2x2 square), and P3 (vertical line in column 1).

- Rules
- i) Neglect zeros.
 - ii) Group can be vertical & horizontal.
 - iii) Overlapping allowed.
 - iv) Mark largest group.
 - v) 2^n cells.

P1

A	B
0	0
1	0

P2

A	B
1	0
1	1

$B' + A \Rightarrow (A + B')$

Q2. $F(A, B, C) = \sum (0, 1, 2, 3, 5)$

	B	00	01	11	10
A	0	1	1	1	1
	1	1	1	0	0

Red lines indicate prime implicants. Blue circles indicate groups of 2 cells.

A	B	C
0	0	0
0	0	1
0	1	1
0	1	0

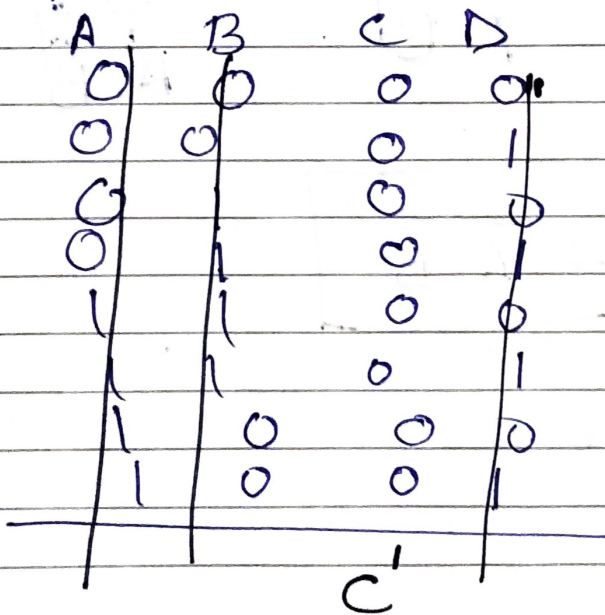
A	B	C
0	0	0
1	0	0
1	1	1
1	0	1

$F = A' + B' + C$

Q3) $F(A, B, C, D) = \sum (0, 1, 4, 5, 8, 9, 12, 15)$

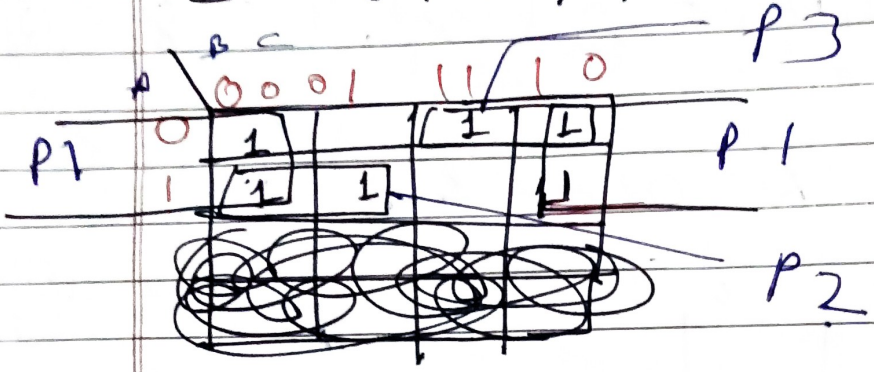
$2^4 = 16$ $A' = 0$ & $A = 1$

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11	1	1		
10	1	1		



Q4

$$\sum m (0, 2, 3, 4, 5, 6)$$



A	B	C
0	0	0
0	1	0
1	0	0
1	1	0

C'

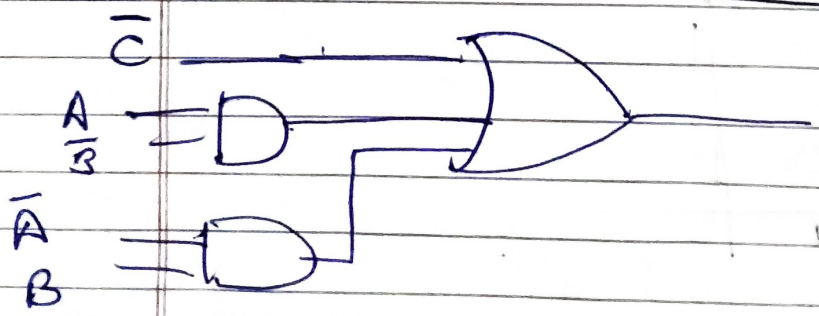
A	B	C
1	0	0
1	0	1

AB'

A	B	C
0	1	1
0	1	0

$A'B$

$f \Rightarrow C' + AB' + \bar{A}B$



Q5 $F(A, B, C, D) = \sum (0, 2, 5, 7, 9, 11)$
 $+ d (3, 8, 10, 12, 14)$

		RS				
	AB	CD	00	01	11	10
00						1
01			1	1		
11		X				X
10		X	1	1		X

\overline{PQ} or \overline{AB}
 $P\overline{Q}$ or $A\overline{B}$

$\begin{array}{r} 0000 \\ 0010 \\ 1000 \\ 1010 \\ \hline \overline{B}\overline{D} \end{array}$	$\begin{array}{r} 0101 \\ 0111 \\ \hline \overline{A}B \cdot D \end{array}$	$\begin{array}{r} 1100 \\ 1110 \\ 1000 \\ 1001 \\ 1011 \\ 1011 \\ \hline A \end{array}$
--	---	---

$\Rightarrow \overline{B}\overline{D} + \overline{A}B \cdot D + A$