COURSE NAME – EM FIELD

SUBJECT COORDINATOR- DR. GAURAV KUMAR BHARTI

Course Name: EM Fields   Code: EC 313

Review of vector algebra, Electric and Magnetic field overview and applications, Maxwell's equations for static and time varying field, boundary conditions for conductor and dielectric. Wave equations for free space, uniform plane waves, linear elliptical and circular polarization, wave equations for conducting medium, wave propagation in conductors and dielectric, depth of penetration, reflection and refraction of plane waves by conductor and dielectric, Poynting vector and flow of power, wave between parallel planes, concept of TE, TM & TEM waves.

Text/ Reference Books:

1. Elements of Electromagnetics — Mathew N.O. Sadiku,
2. Engineering Electromagnetics — W.H. Hayt,

Vector Algebra

- Vectors algebra is the branch of algebra that involves operations on vectors.
- Vectors are quantities that have both magnitude and direction so normal operations are not performed on the vectors.
- We can add, subtract, and multiply vector quantities using special vector algebra rules.
- Vectors can be easily represented in 2-D or 3-D spaces.
- Vector algebra has various applications it is used in solving various problems in mathematics and physics, engineering, and various other fields.

What Is Vector Algebra

Vector algebra is the type of Algebra that is used to perform various algebraic operations on vectors. As we know vectors are quantities that have both magnitude and direction whereas scalar quantities only have magnitude and no direction

Representation of Vectors

- Vectors are represented by taking an arrow above the quantity, i.e. force vector is represented as $\vec{F}$ where the arrow above F represents that it is a vector quantity.

- Vectors can also be represented by taking their respective magnitude in x, y, and z-directions respectively. The x-direction is shown using $\hat{\imath}$, similarly, the y-direction is shown using $\hat{\jmath}$and the z-direction is shown using $\hat{k}$.

Now the vector A is represented as,

$$\tilde{\mathbf{A}} = \mathbf{x}\hat{\mathbf{i}} + \mathbf{y}\hat{\mathbf{j}} + \mathbf{z}\hat{\mathbf{k}}$$

➢ The point where the vector start is called the tail of the vector and the endpoint of the vector is called the head of the vector. We can also denote the vector as the coordinate point in 3-Dimensions.

The basis vectors are denoted as
- $e_1 = (1,0,0)$
- $e_2 = (0,1,0)$
- $e_3 = (0,0,1)$

Magnitude of Vectors

- The magnitude of a vector represents the strength of the vector.
- We can calculate the magnitude of the vector by taking the square root of the sum of the squares of each component in the x, y, and z directions.
- The magnitude of a vector is calculated by taking the square root of the sum of the square of the components of the vector in the x, y, and z directions.
- For any vector $\vec{A} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$, the magnitude of the vector is represented as |A| and its value is

$$|A| = \sqrt{(a^2+b^2+c^2)}$$

Components of Vectors

- A vector can be easily broken into its two components which represent the value of the vector in perpendicular dimensions. In a 2-D coordinate system, we can easily break the vector into two components namely the x-component and y-component.

- For any vector $\vec{A}$ its,

- x-components is Ax and its value is $A_x = A \cos\theta$

- Y-components is Ay and its value is $A_y = A \sin\theta$

- where $\boldsymbol{\theta}$ is the angle formed by the vector with the positive x-axis. Also, the magnitude of the vector A is calculated using the formula,

$$|A| = \sqrt{[(A_x)^2 + (A_y)^2]}$$

Angle Between Two Vectors

- If two vectors in the 2-D plane intersect each other then the angle between them can easily be calculated using the dot product of the vector formula.

- We know that for two vectors vector a, and vector b their dot product is given by,

$$\vec{A} \cdot \vec{B} = |a|.|b|. \cos \theta$$

Types of Vectors

Vectors can be classified into different categories on the basis of their magnitude and direction. The various types of vectors are listed below:

- Zero Vector
- Unit Vector
- Equal Vector
- Negative Vector
- Co-Initial Vectors
- Collinear Vectors
- Parallel Vectors
- Orthogonal Vectors

Operations in Vector Algebra

We can perform various operations in Vector Algebra by taking a geometrical approach or by taking a coordinate system approach.

Various operations in vector algebra are,

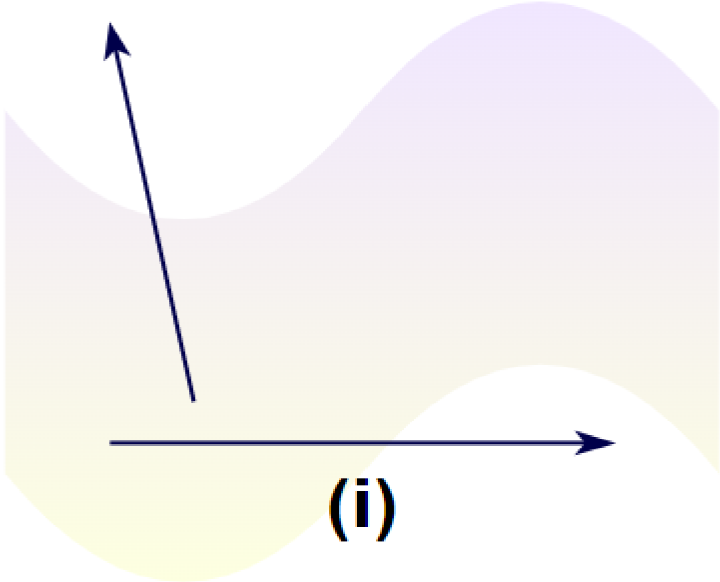
- Addition of Vectors
- Subtraction of Vectors
- Multiplication of Vectors by Scalar
- Scalar Triple Product of Vectors
- Multiplication of Vectors

Triangle Law of Vector Addition

- The triangle law of vector addition states that if two sides of any triangle represent the two vectors that are on a body in the same order as the side of the triangle, then the third side of the triangle represents the resultant vector.

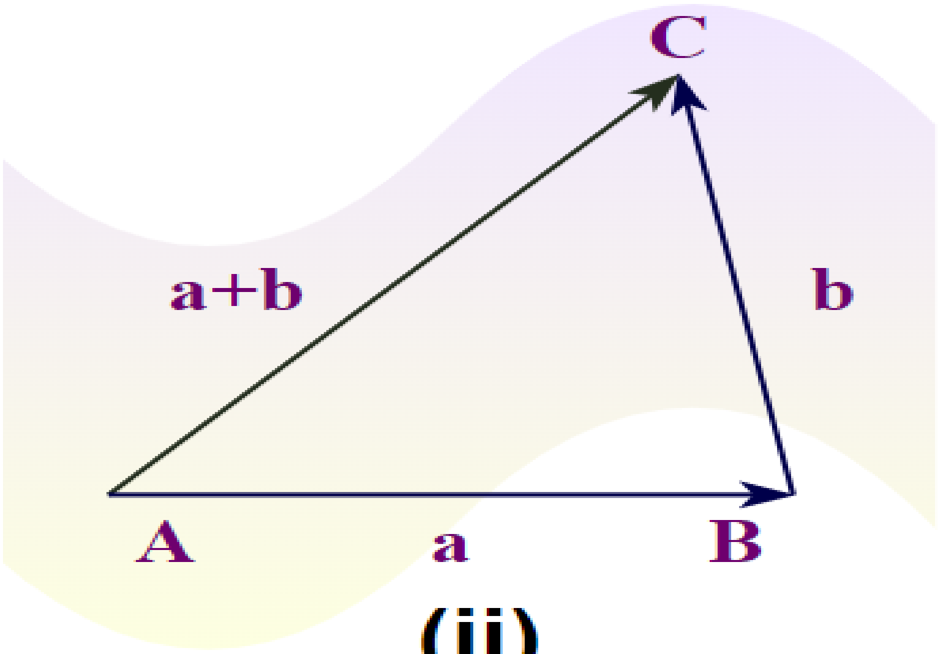
- In general, for two vectors $\vec{A}$ and $\vec{B}$ their addition is done such that the initial point of one vector coincides with the terminal points of the other.

- In the figure, we have two vectors $\vec{A}$ and $\vec{B}$ given



(i)

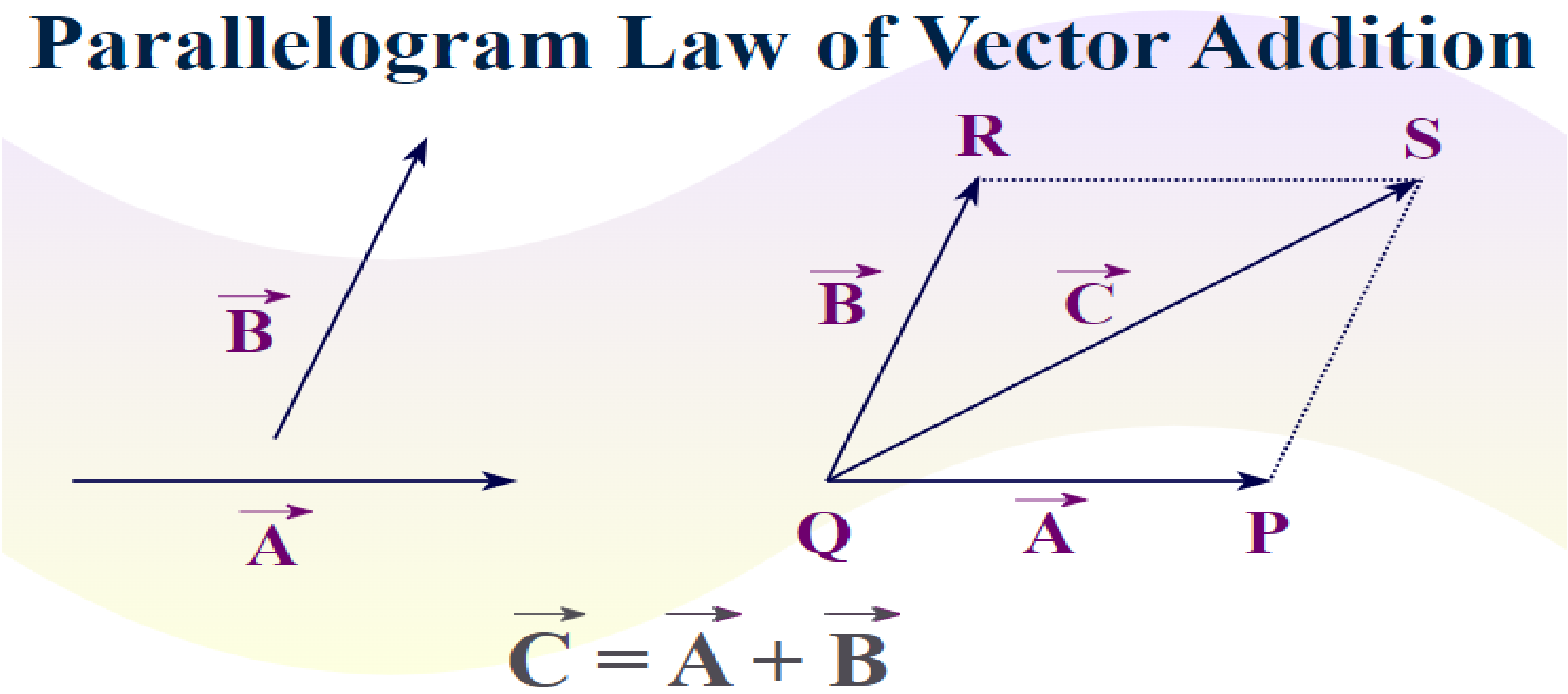
Now the vector b is shifted without changing its direction and magnitude such that now the initial point of vector $\vec{B}$ lies on the endpoint of vector a as shown in the figure,



(ii)

Parallelogram Law of Vector Addition

Parallelogram law of vector addition states that if the adjacent side of the parallelogram represents two vectors then the diagonal starting from the same initial point represents the resultant of the vector.



Properties of Vector Addition

Various properties of vector addition are

Property 1: Vector addition follows commutative property. For two vectors $\vec{A}$ and $\vec{B}$.

$$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}$$

Property 2: Vector addition of three vectors follows the associative property.

$$\overrightarrow{(\boldsymbol{A} + \vec{\boldsymbol{B}})} + \vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + (\vec{\boldsymbol{B}} + \vec{\boldsymbol{C}})$$

Applications of Vector Algebra

- Vector algebra is widely used in various fields such as Mathematics, Engineering, Physics, and others.
- Various physical quantities encountered by us in real life such as force, acceleration, velocity, and others are vector quantities and we use vector algebra to define and operate on those quantities.

Some of the important applications of vector algebra are

- Vector algebra is used to study electromagnetic fields, gravitational fields, fluid flow, and other topics in engineering and physics.
- Differential Equation and Partial Differential Equations are solved using vector algebra.
- The resultant of the force acting on the body is calculated using vector algebra.
- Vector Algebra is used to find equipotential surfaces.

Scalar	Vector
1. The quantities which has only magnitude, and no direction, are scalar quantities	1. The quantities which has magnitude and direction are vector quantities
2. The scalar quantities are one dimensional quantities	2. The vector quantities are more than one dimensional quantities
3. The scalar can be divided with another scalar quantities	3. The vector can not be divided with another vector quantities
4. Number (Magnitude) and unit is only required to represent scalar quantities	4. Number (Magnitude), direction (Unit vector) and unit is required to represent vector quantities

Basics of Vector

- Representation of vector

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{A} = (A_x, A_y, A_z)$$

- Magnitude of vector

$$A = |\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2}$$

- Unit vector

$$\widehat{i_A} = \frac{\vec{A}}{|\vec{A}|}$$

Dot Product

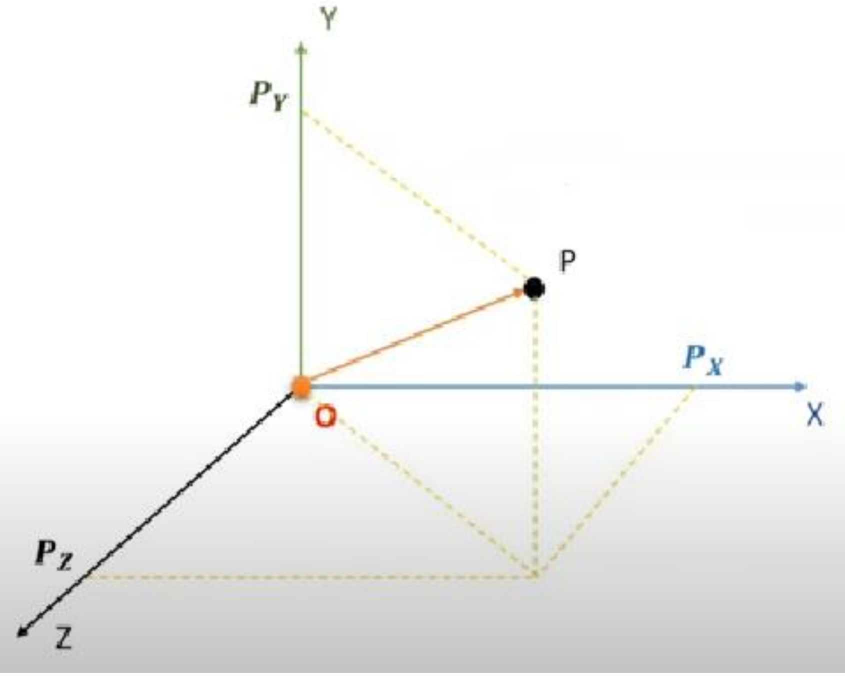
- Dot product is scalar product of two vector, so Resultant of dot product is scalar quantity.
- If we have two vectors given by $\vec{A}$ and $\vec{B}$,

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad \vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

- Then Dot product is given by

$$Y = \vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

Representation of cartesian coordinate system



$$\vec{P} = P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k}$$

Projection of vector

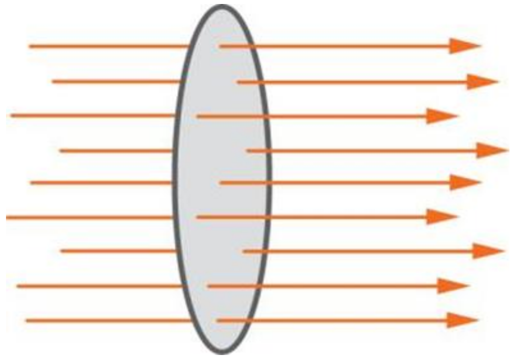
- **Projection of vector $\vec{A}$ on $\vec{B}$ is A cos θ, where θ is angle between $\vec{A}$ and $\vec{B}$.**
- **Projection of vector $\vec{B}$ on $\vec{A}$ is B cos θ, where θ is angle between $\vec{A}$ and $\vec{B}$.**

$$\vec{A}.\vec{B} = AB\cos\theta \Rightarrow A\cos\theta = \frac{\vec{A}.\vec{B}}{B}$$

$$\vec{A}.\vec{B} = AB\cos\theta \Rightarrow B\cos\theta = \frac{\vec{A}.\vec{B}}{A}$$

Permanent magnets

- Magnetic field lines start and finish at poles. Physicists picture this as a ‘flow’ in magnetic circuit.

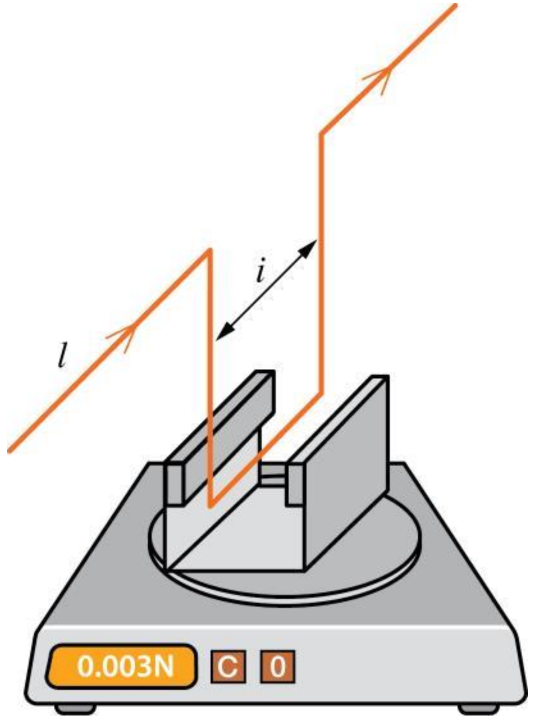


- magnetic flux ϕ(phi), unit Weber
- magnetic flux density B, unit Weber m^{-2} or Tesla
- Carl Gauss & Wilhelm Weber investigated geomagnetism in 1830s, made accurate measurements of magnetic declination and inclination, built the first electromagnetic telegraph.

Defining magnetic flux density

Fleming's left-hand rule:
Force on the wire is perpendicular to both *I* and *B*.

$$B = \frac{F}{Il}$$



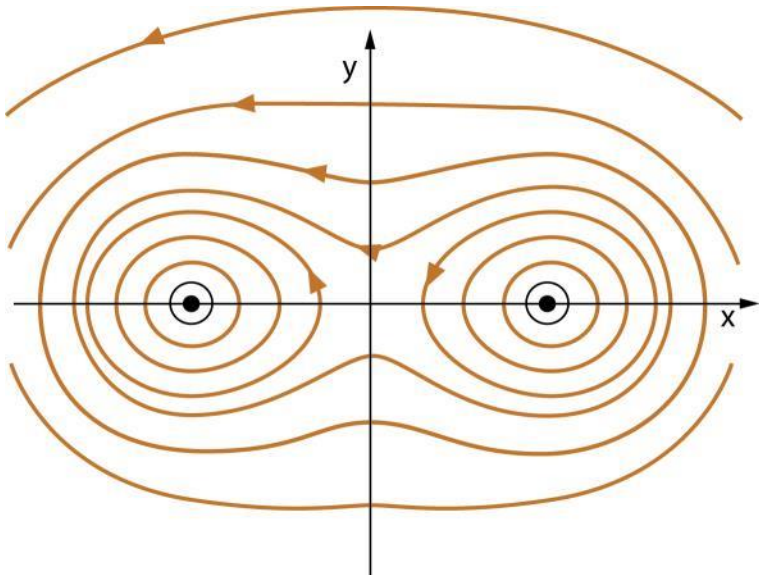
Magnetic fields near currents

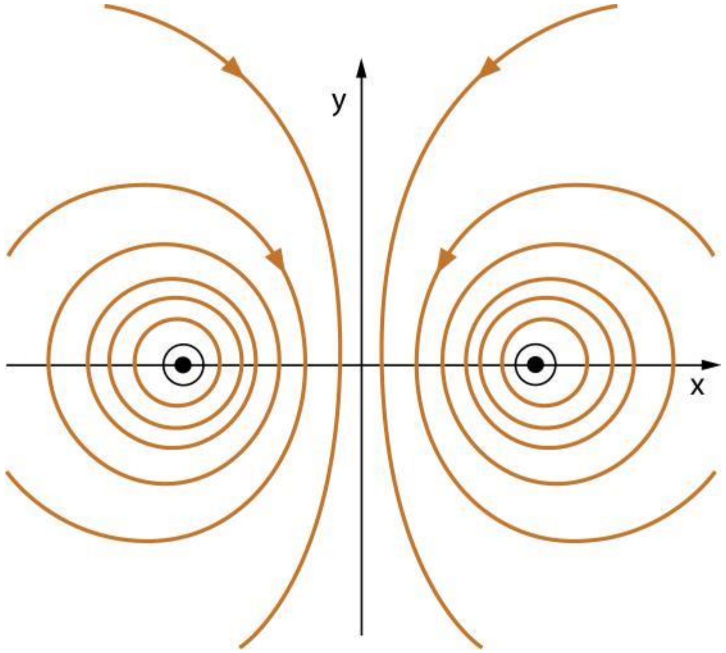
- long straight wire $$B = \frac{\mu_0 I}{2\pi r}$$

long solenoid, N turns and length l $$B = \mu_0 \frac{N}{l} I$$

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{NA}^{-2}$ is the permeability of free space

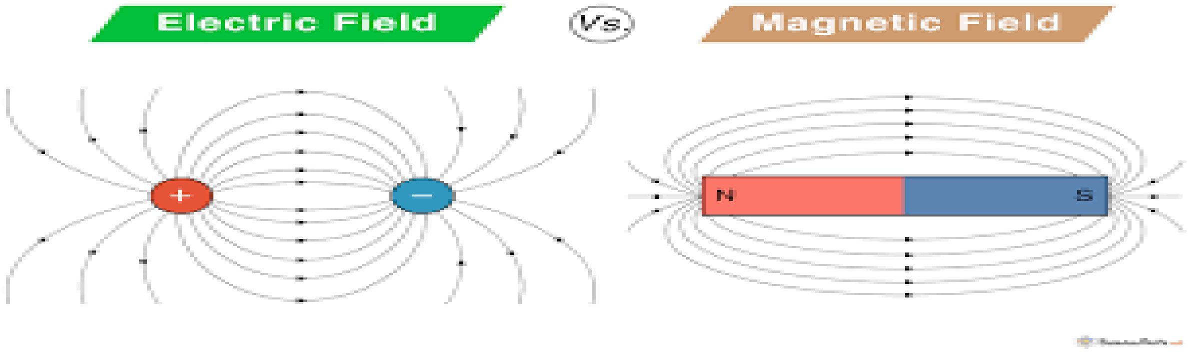
Forces on parallel currents

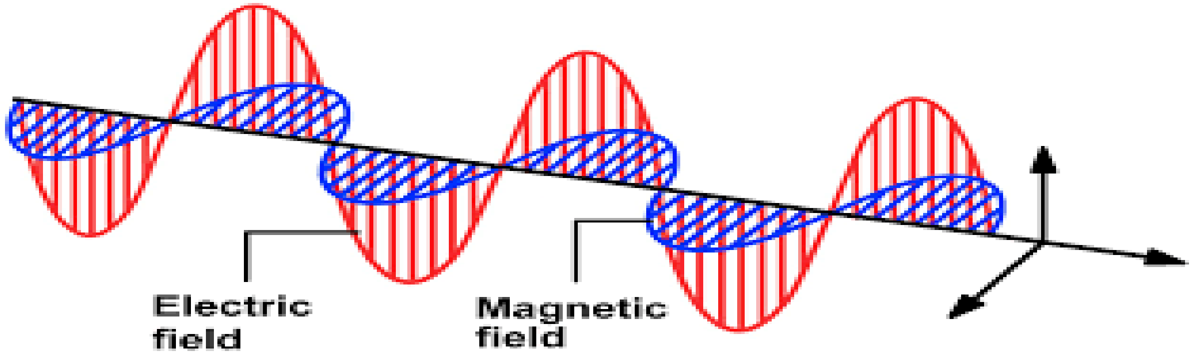




Difference between Electric Field vs Magnetic Field

Electric Field	Magnetic Field
Measured as newton per coulomb, volt per metre.	Measured as gauss or tesla.
Proportional to the electric charge.	Proportional to the speed of electric charge.
It is perpendicular to the magnetic field.	It is perpendicular to the electric field.
An electric field is measured using an electrometer.	The magnetic field is measured using the magnetometer.


Electric Field
Vs.
Magnetic Field
+
−
N
S


Electric
field
Magnetic
field

- An electric field is defined mathematically as a vector field that can be associated with each point in space, the force per unit charge exerted on a positive test charge at rest at that point.

- The formula of the electric field is given as,

- $E = F/Q$

- Where,

- E is the electric field.
- F is the force.
- Q is the charge.

- The direction of the field is taken as the direction of the force which is exerted on the positive charge. The electric field is radially outwards from the positive charge and radially towards the negative point charge.