POYNTING VECTOR
&
EQUATIONS

Poynting vector:

Poynting Vector 'S' is defined as the cross product of the vectors E & H.

The direction of power flow at any point is normal to both E & H vectors.

The Unit is watts/m²

Poynting vector is $\vec{S} = \vec{E} \wedge \vec{H} = \left(E_y H_z, 0, -E_y H_x\right)$

Time-averaged: $\langle \vec{S} \rangle = \frac{1}{2}(0,0,1)\frac{kA^2}{\omega\mu}\sin^2\frac{n\pi x}{a}$

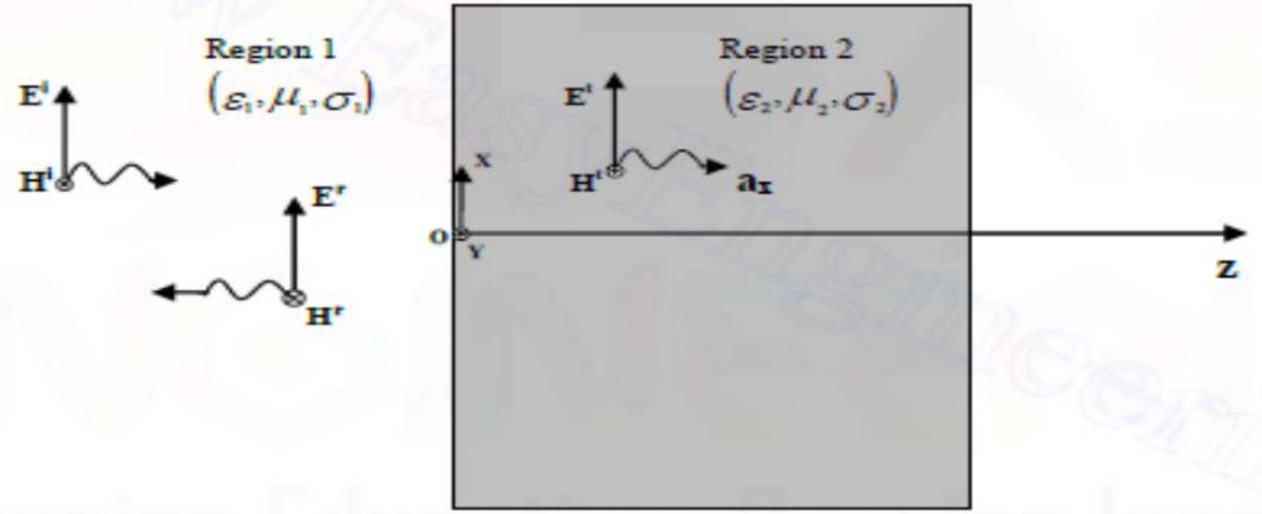
Integrate over *x*: $\langle S_z \rangle = \frac{1}{4}\frac{akA^2}{\omega\mu}$

Total e/m energy density

$$W = \frac{1}{4}\varepsilon A^2 a$$

So energy is transported at a rate: $\frac{\langle S_z \rangle}{W_e + W_m} = \frac{k}{\omega\varepsilon\mu} = v_g$

Electromagnetic energy is transported down the waveguide with the group velocity



Normal Incidence Plane Wave Reflection and Transmissions at Plane Boundary Between Two Conductive Media

The electric and magnetic fields related to the incident wave are given by the following:

$$\hat{E}_x^i = \hat{E}_{m1}^+ e^{-\hat{\gamma}_1 z}$$

$$\hat{H}_y^i = \frac{\hat{E}_{m1}^+}{\hat{\eta}_1} e^{-\hat{\gamma}_1 z}$$

* Note: (i) incident, (m_1) medium 1, (γ_1) propagation constant in region 1, ($\eta 1$) wave impedance in region 1, (z) direction of propagating wave

$$\gamma = \alpha + j\beta$$

With $$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)}$$

The wave impedance as defined in chapter 2 as the ratio between the electric and magnetic fields is

$$\frac{\hat{E}_x}{\hat{H}_y} = \hat{\eta} = \frac{\mu}{\left(\varepsilon - j\frac{\sigma}{\omega}\right)} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}} e^{j\frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)}$$

Poynting Theorem from Maxwell's Equations:

Maxwell's equation in the point form is

Equation (1) $$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Taking dot product with 'E' on both sides

$$E \cdot (\nabla \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t}$$

From vector identity,

$$\nabla \cdot (E \times H) = H \cdot (\nabla \times E) - E(\nabla \times H)$$

Equation (2)

$$E \cdot (\nabla \times H) = H \cdot (\nabla \times E) - \nabla \cdot (E \times H)$$

Substituting equation (2) in (1)
Equation (3) is

$$H \cdot (\nabla \times E) - \nabla \cdot (E \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t}$$

From Maxwell's Equation $(\nabla \times E) = -\frac{\partial B}{\partial t}$

$$H\left(\frac{-\partial B}{\partial t}\right) - \nabla \cdot (E \times H) = E \cdot J + E \cdot \frac{\partial D}{\partial t}$$

Equation (4)

$$\nabla \cdot (E \times H) = -E \cdot J - H \cdot \left(\frac{\partial B}{\partial t}\right) - E \cdot \frac{\partial D}{\partial t}$$

Substituting $J = \sigma E, D = \varepsilon E\ , B = \mu H$

in Equation (4)

$$\nabla \cdot (E \times H) = -\sigma E^2 - E \cdot \frac{\partial(\varepsilon E)}{\partial t} - H \frac{\partial(\mu H)}{\partial t}$$

$$\nabla \cdot (E \times H) = -\sigma E^2 - \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2\right) - \frac{\partial}{\partial t}(\frac{1}{2}\mu H^2)$$

Integrating throughout the volume

$$\int_{vol} \nabla \cdot (E \times H) dv = -\int_{vol} \sigma E^2 dv - \int_{vol} \frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) dv$$

Total Power — Power dissipation — Rate of energy stored

Using divergence theorem

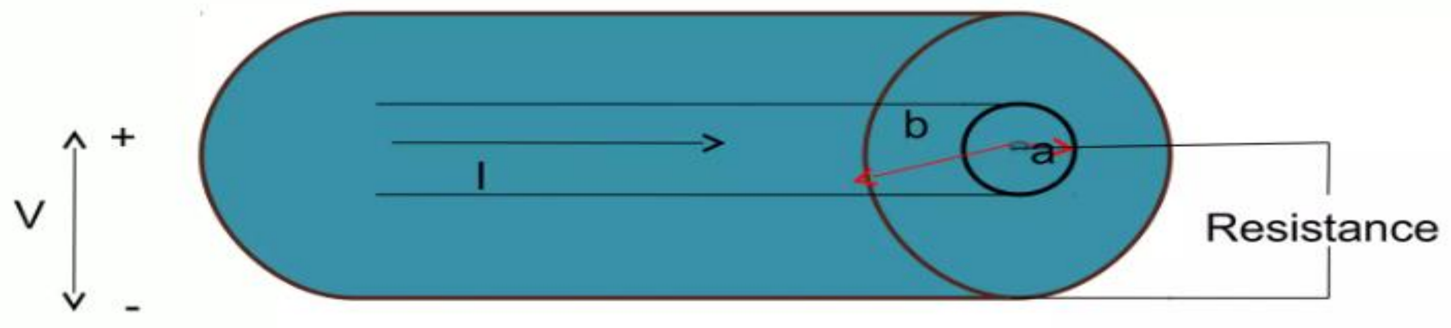
$$\int_S D.ds = \int_{vol} \nabla \cdot D \, dv$$

$$\int_{vol} \nabla \cdot (E \times H) dv = \int_S (E \times H) \cdot dS$$

Power flow in a co-axial cable

Consider a co-axial cable which has a dc voltage ‘V’ between the conductors and a steady current I flowing in the inner and outer conductors.

The radius of inner and outer conductor are ‘a’ and ‘b’ respectively.



By ampere's Law:

$$\int H \cdot dL = I$$

$$\int dL =$$ Circumference of circular path between a and b =2πr

$$H \cdot (2\pi r) = I$$

$$H = \frac{I}{2\pi r} \quad a < r < b$$

E due to an infinitely long conductor

$$E = \frac{\lambda}{2\pi\varepsilon r} \qquad \text{Equation (1)}$$

Where λ is the charge density.

The potential difference between the conductors is

$$V = \frac{\lambda}{2\pi\epsilon} In\left(\frac{b}{a}\right) \qquad \text{Equation (2)}$$

E in terms of V from (1) and (2) is

$$E = \frac{V}{In\left(\frac{b}{a}\right)r}$$

Power density $P = E \times H$

Since E and H are always perpendicular to each other

$$P = E \cdot H$$

$$P = \frac{V}{In\left(\frac{b}{a}\right)r} \cdot \frac{I}{2\pi r}$$

The total power will be given by the integration of power density P over any cross section surface.

Let the elemental surface already be 2πrdr

Total power $$W = \int \frac{V}{In\left(\frac{b}{a}\right) r} \cdot \frac{I}{2\pi r} (2\pi r)\, dr$$

$$W = \frac{V}{In\left(\frac{b}{a}\right)} I \int_a^b \frac{1}{r}\, dr$$

$$W = \frac{V}{In\left(\frac{b}{a}\right)} \cdot I\left(In\frac{b}{a}\right)$$

$$W = VI$$

i.e. The power flow along the cable is the product of V and I

THANK YOU

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