COURSE NAME – EM FIELD

SUBJECT COORDINATOR- DR. GAURAV KUMAR BHARTI

Electromagnetic field

Four vector quantities

$\mathscr{E}$	electric field	[Volt/meter]
$\mathscr{D}$	electric flux density	[Coulomb/meter^2]
$\mathscr{H}$	magnetic field	[Amp/meter]
$\mathscr{B}$	magnetic flux density	[Weber/meter^2] or [Tesla]

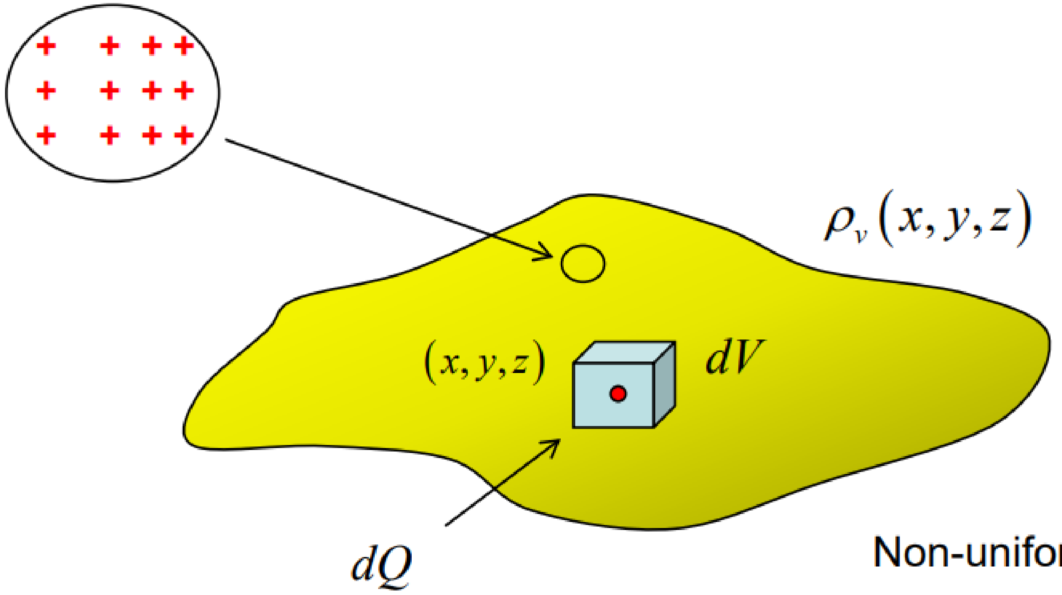
Each are functions of space and time
e.g. $\mathscr{E}(x,y,z,t)$

Reminder:
The Handscript SF font is used to denote time-varying vectors.

$\mathscr{J}$	electric current density	[Amp/meter^2]
ρ_v	electric charge density	[Coulomb/meter^3]

Charge density

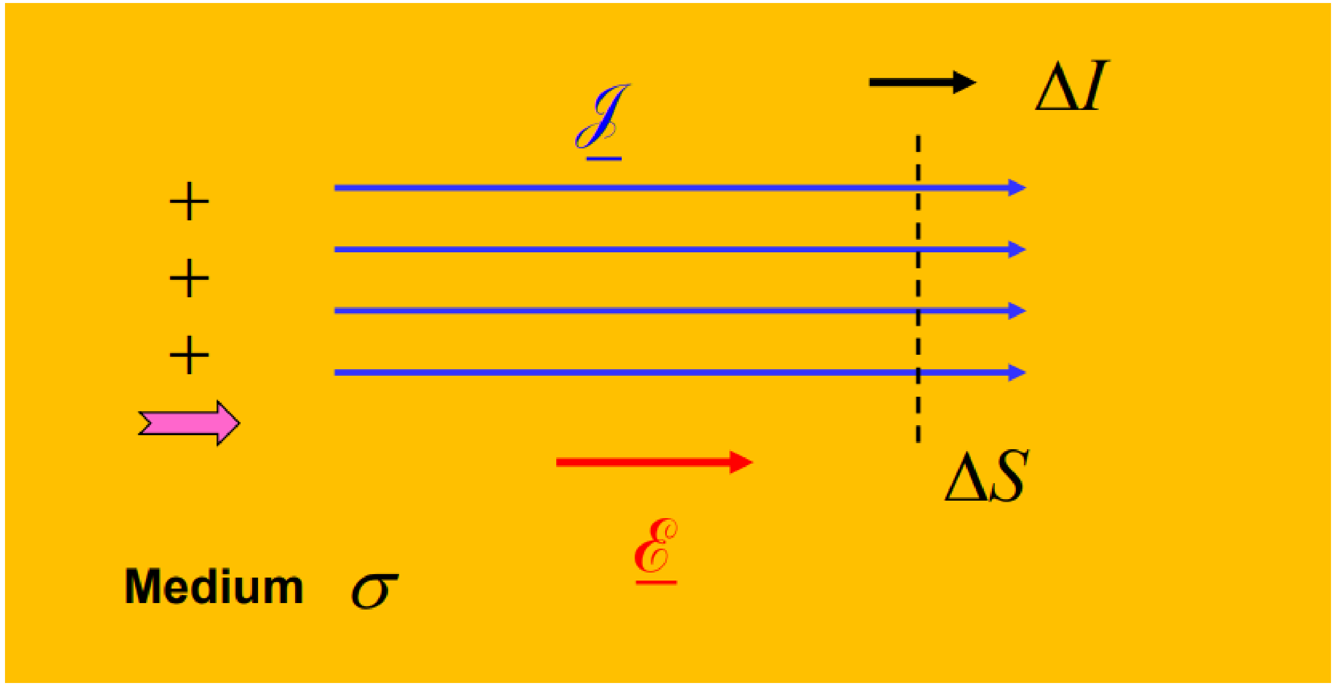
$$\rho_v(x,y,z) = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$



Non-uniform cloud of charge density

Current density vector

$\underline{\mathcal{J}}$ = current density vector $\left[\mathrm{A/m^2}\right]$



$$\Delta I = \left|\underline{\mathcal{J}}\right| \Delta S$$

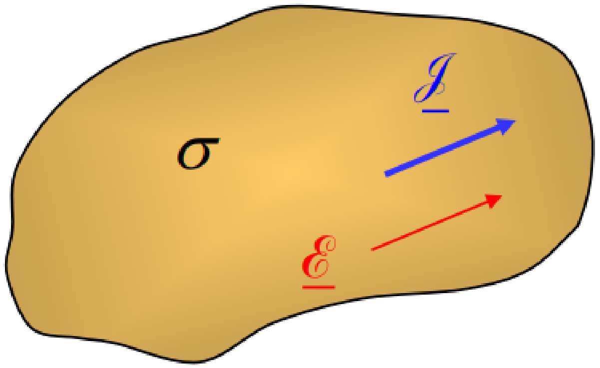
Current flow is defined to be in the direction that positive charges move in.

Note: If negative charges are moving, we can pretend that positive charges are moving in the opposite direction.

Current density vector

Ohm's law

$$\underline{\mathcal{J}} = \sigma \underline{\mathcal{E}}$$



Material	σ [S/m]
Silver	6.3×10^{7}
Copper	6.0×10^{7}
Copper (annealed)	5.8×10^{7}
Gold	4.1×10^{7}
Aluminum	3.5×10^{7}
Zinc	1.7×10^{7}
Brass	1.6×10^{7}
Nickel	1.4×10^{7}
Iron	1.0×10^{7}
Tin	9.2×10^{6}
Steel (carbon)	7.0×10^{6}
Steel (stainless)	1.5×10^{6}

Conductor

- In physics and electrical engineering, a **conductor** is an object or type of material that allows the flow of charge (electric current) in one or more directions. Materials made of **metal** are common electrical conductors. The flow of negatively charged electrons generates electric current, positively charged **holes**, and positive or negative ions in some cases.

- In order for current to flow within a closed electrical circuit, one charged particle does not need to travel from the component producing the current (the current source) to those consuming it (the loads).

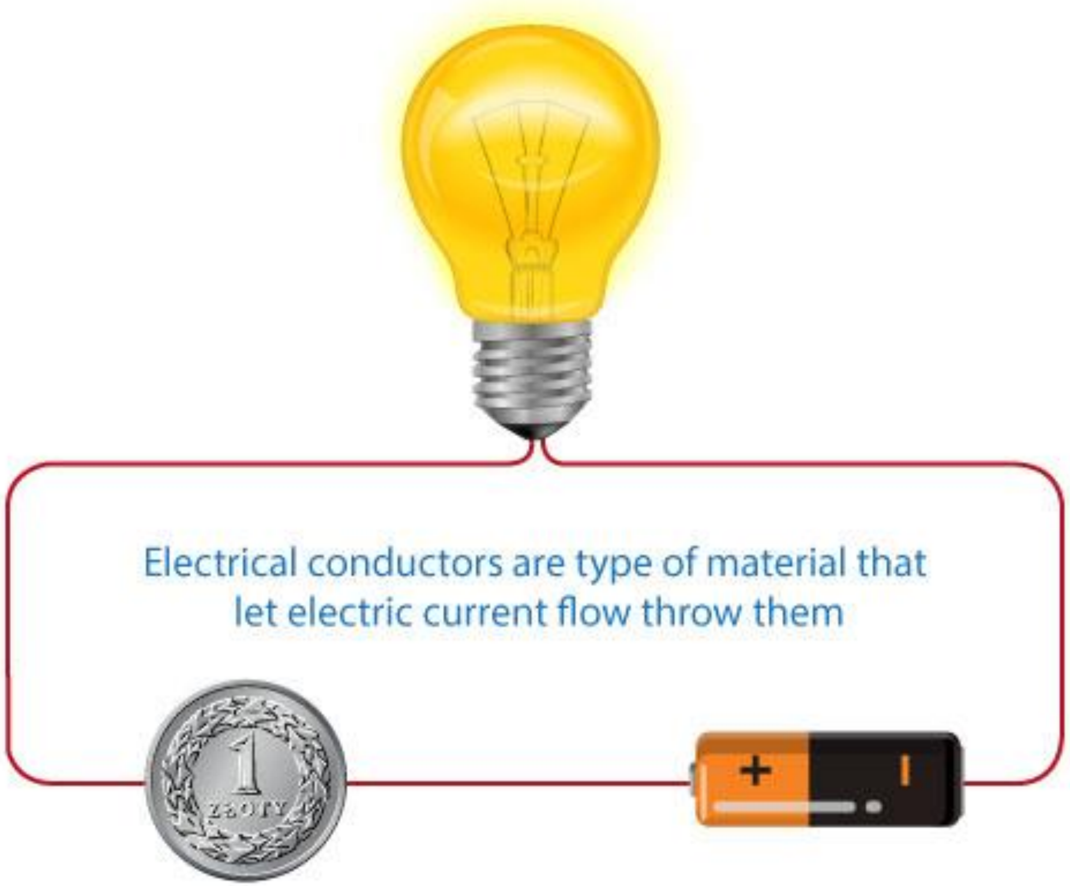
- Essentially what is occurring is a long chain of momentum transfer between mobile charge carriers; the **Drude model** of conduction describes this process more rigorously.

- This momentum transfer model makes metal an ideal choice for a conductor; metals, characteristically, possess a delocalized sea of electrons which gives the electrons enough mobility to collide and thus affect a momentum transfer.

CONDUCTORS

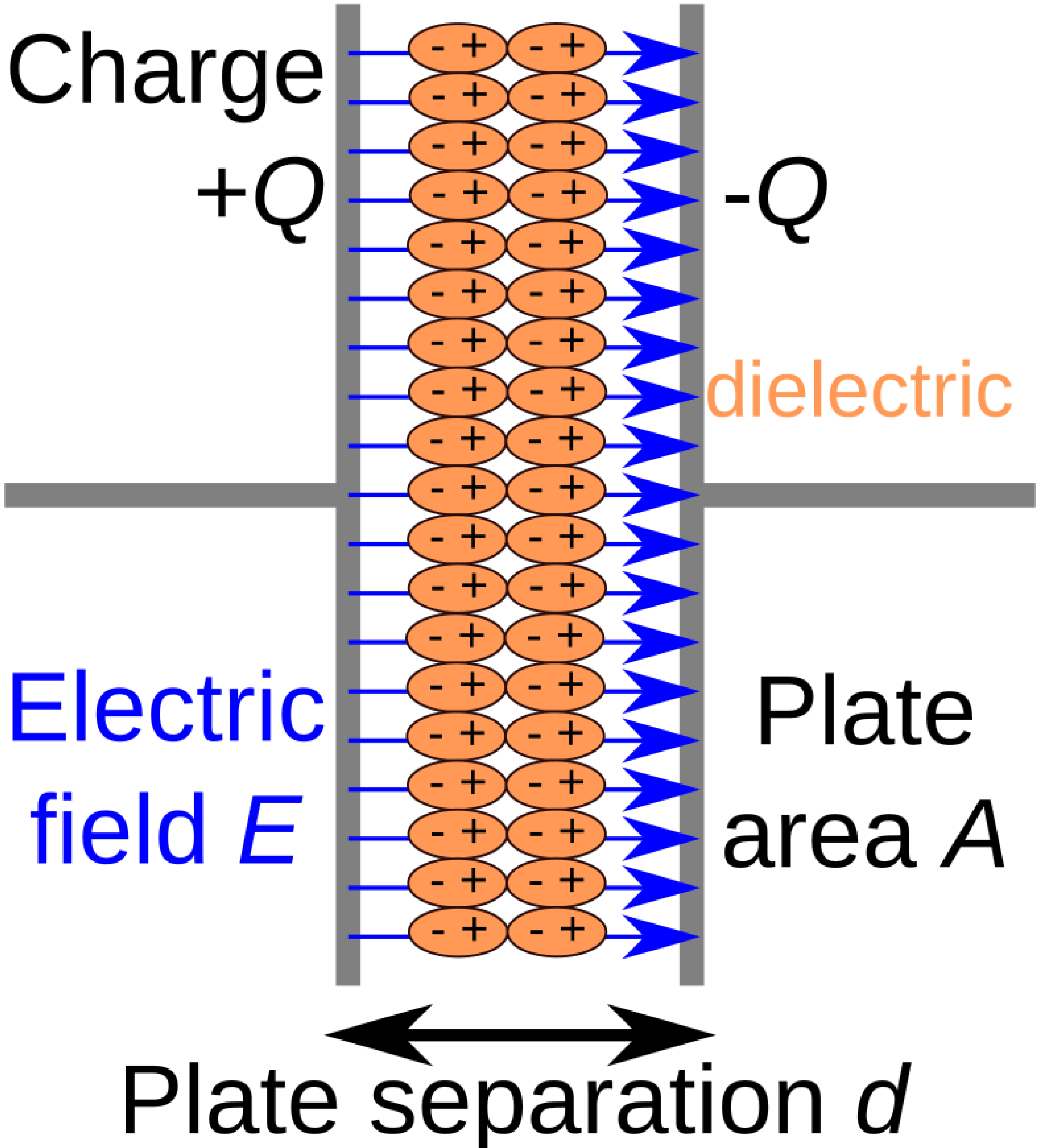

Copper
Gold
Sea water
Steel
Electrical Conductor Examples
Silver
Lemon Juice

Electrical Conductor



Dielectric

- In **electromagnetism**, a dielectric (or dielectric medium) is an **electrical insulator** that can be **polarised** by an applied **electric field**.
- When a dielectric material is placed in an electric field, **electric charges** do not flow through the material as they do in an **electrical conductor**, because they have no loosely bound, or free, electrons that may drift through the material, but instead they shift, only slightly, from their average equilibrium positions, causing dielectric polarisation.
- Because of **polarisation**, positive charges are displaced in the direction of the field and negative charges shift in the direction opposite to the field.
- This creates an internal electric field that reduces the overall field within the dielectric itself. If a dielectric is composed of weakly **bonded** molecules, those molecules not only become polarised, but also reorient so that their **symmetry axes** align to the field.


Charge
+Q
-Q
dielectric
Electric
field E
Plate
area A
Plate separation d

DIFFERENCE BETWEEN DIELECTRIC AND CONDUCTOR

Dielectrics

- Electric polarization dominates
- Small number of free charges ~10^10 to ~10^16 1/m^3
- Small bulk conductivity ~10^-9 to ~10^-17 1/Ohm*m (large resistivity)
- Conductivity increases with the temperature

C.A. Balanis, Advanced engineering electromagnetics, 2012
I. S Rez, Y.M. Poplavko, Dielectrics (in Russian), 1989

Semi-metals
Semiconductors

Conductors

- Almost no electric polarization up to ~10^16 Hz (shielding)
- Large number of free charges ~10^27 to ~10^29 1/m^3
- Large bulk conductivity ~10^6 to ~10^8 1/Ohm*m (small resistivity)
- Conductivity decreases with the temperature

Dielectric materials

Polar molecules

(i) Permanent electric dipole

(ii) Its polarisation is temperature dependent

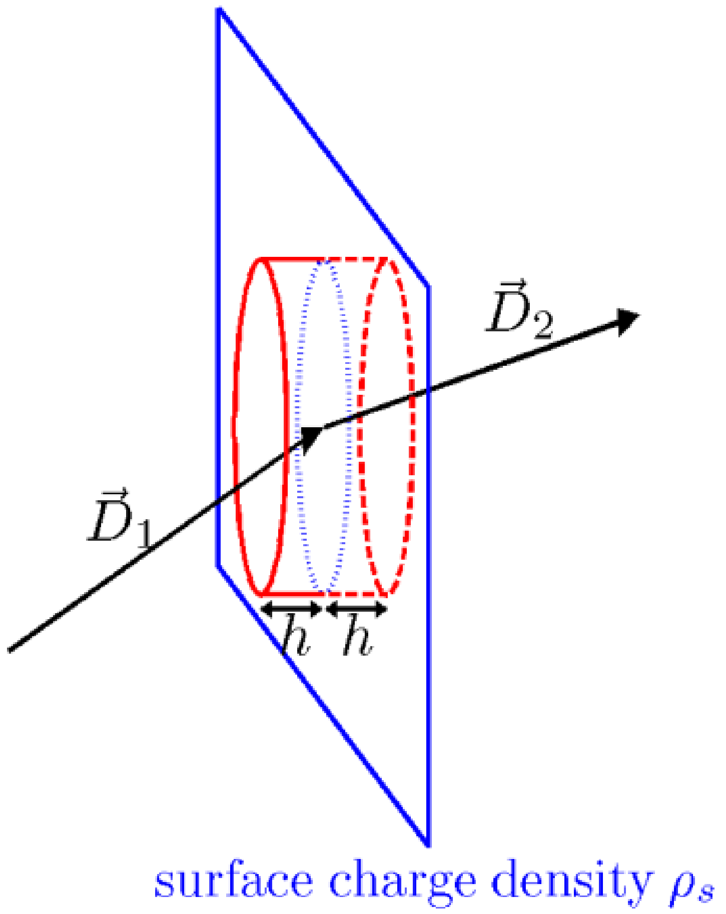
Non-polar molecules

(i) Induced electric dipole

(ii) Its polarisation independent of temperature

Boundary conditions: normal component of D

- Consider the electric displacement at a boundary between two different materials.
- We need make no assumptions about the properties of the materials.
- To be completely general, we will assume that there is some surface charge density ρs (charge per unit area) on the boundary.
- We will apply Gauss' theorem to the divergence of the electric displacement, integrated over a pillbox crossing the boundary.



WAVE EQUATION

Electromagnetic Waves

If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportional to the space separation from the first location, then the group of phenomena constitute a wave.

In general, Waves are means of transporting energy or information.

A wave is a carrier of energy or information and is a function of both time and space.

Typical examples of Electro Magnetic (EM) waves include radio waves, TV signals, radar beams and light rays.

All forms of EM energy share three fundamental characteristics:

(i) They all travel with high velocity;

(ii) In traveling, they assume all the properties of waves; and

(iii) They radiate outward from a source.

Maxwell predicted the existence of EM waves and established it through well known Maxwell's equations.

The same EM waves were investigated by Heinrich Hertz. Hertz conducted several experiments on EM waves and he succeeded in generating and detecting radio waves. These radio waves are also called Hertzian waves.

Applications of EM waves

(i) They have a wide range of applications in all types of communications like Television, satellite, wireless, cellular and mobile communications etc.

(ii) EM waves are also used in radiation therapy and microwave ovens etc.

(iii) In all types of radars like doppler radar, airport surveillance radar, weather forcasting radar, remote sensing radar etc.

Polarization

- Maxwell (in1865) give the theory of EM wave and tell that light is an EM wave because it resemble the properties of EM wave.
- Speed ($3*10^8$) m/s
- Does not require any medium for propagation

- EM wave are created by accelerating charge and vibrating charge also create EM wave

- EM waves are Transvers in nature, Transvers means electric field vector, magnetic field vector and direction of propagation are perpendicular to each other.

Polarization

- **Polarization** (also **polarisation**) is a property of transverse waves which specifies the geometrical orientation of the oscillations.
- In a transverse wave, the direction of the oscillation is perpendicular to the direction of motion of the wave.
- A simple example of a polarized transverse wave is vibrations traveling along a taut string, for example, in a musical instrument like a guitar string. Depending on how the string is plucked, the vibrations can be in a vertical direction, horizontal direction, or at any angle perpendicular to the string.
- In contrast, in longitudinal waves, such as sound waves in a liquid or gas, the displacement of the particles in the oscillation is always in the direction of propagation, so these waves do not exhibit polarization.
- Transverse waves that exhibit polarization include electromagnetic waves such as light and radio waves, gravitational waves, and transverse sound waves (shear waves) in solids.

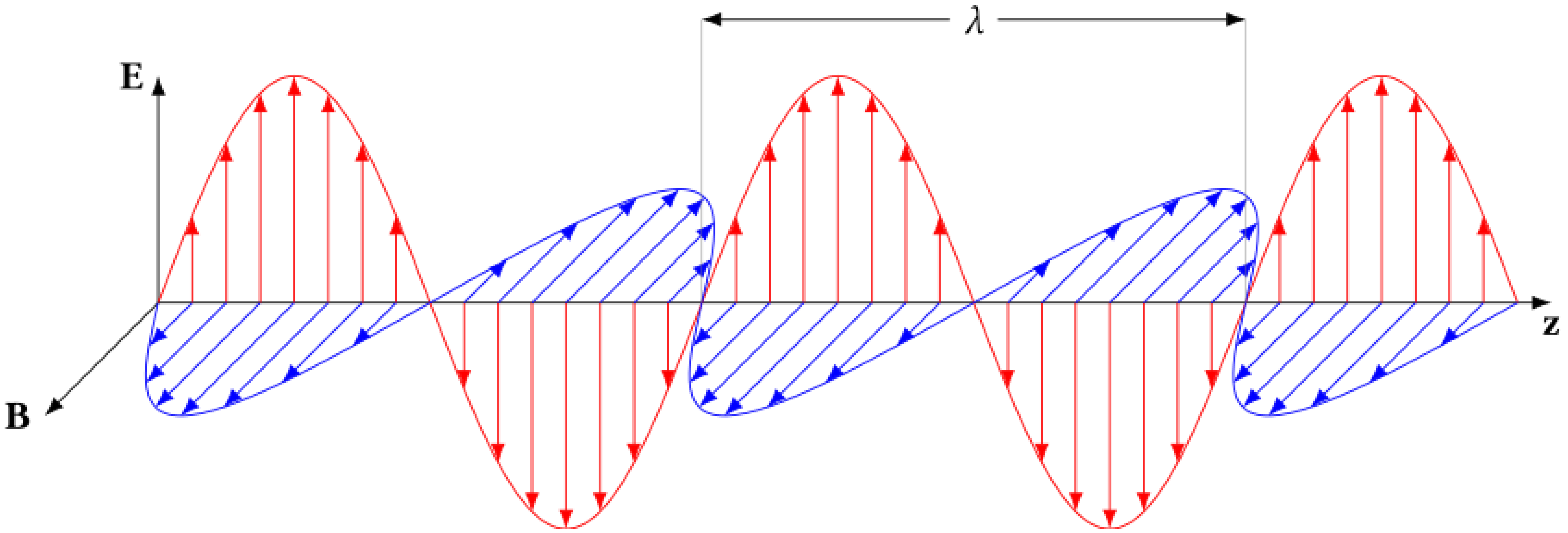
- An electromagnetic wave such as light consists of a coupled oscillating electric field and magnetic field which are always perpendicular to each other; by convention, the "polarization" of electromagnetic waves refers to the direction of the electric field.
- In linear polarization, the fields oscillate in a single direction. In circular or elliptical polarization, the fields rotate at a constant rate in a plane as the wave travels, either in the right-hand or in the left-hand direction.

- Light or other electromagnetic radiation from many sources, such as the sun, flames, and incandescent lamps, consists of short wave trains with an equal mixture of polarizations; this is called *unpolarized light*.

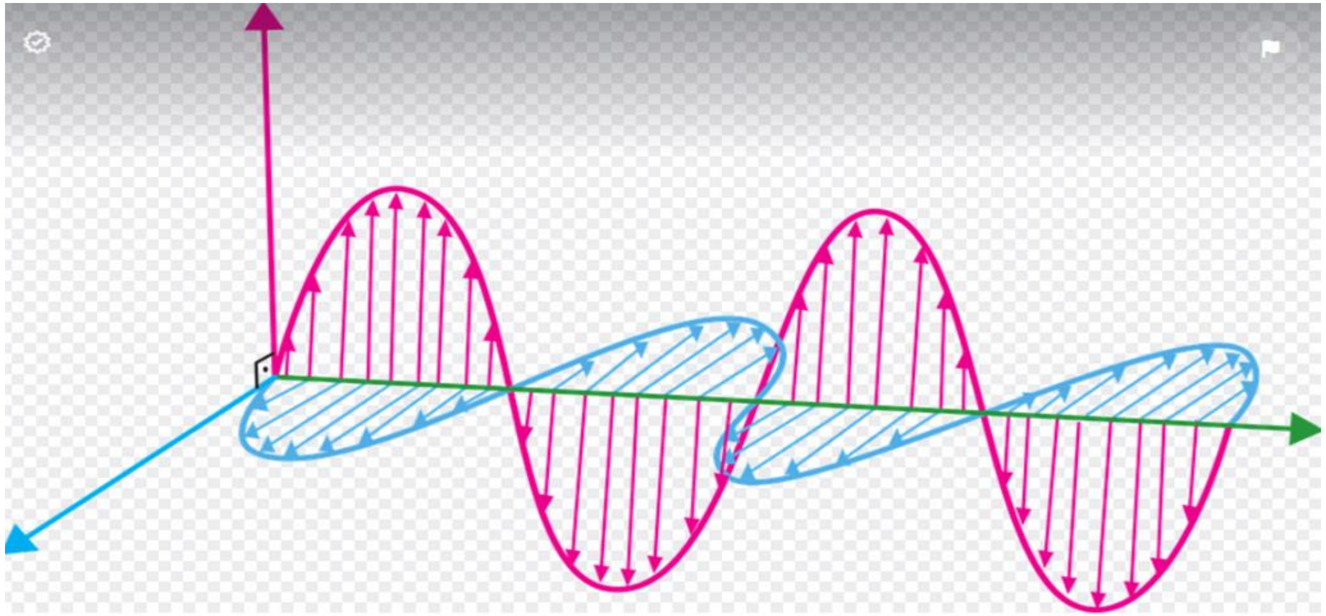
- Polarized light can be produced by passing unpolarized light through a polarizer, which allows waves of only one polarization to pass through. The most common optical materials do not affect the polarization of light, but some materials—those that exhibit birefringence, dichroism, or optical activity—affect light differently depending on its polarization.

- Some of these are used to make polarizing filters.

- Light also becomes partially polarized when it reflects at an angle from a surface.



A "vertically polarized" electromagnetic wave of wavelength λ has its electric field vector **E** (red) oscillating in the vertical direction. The magnetic field **B** (or **H**) is always at right angles to it (blue), and both are perpendicular to the direction of propagation (**z**).



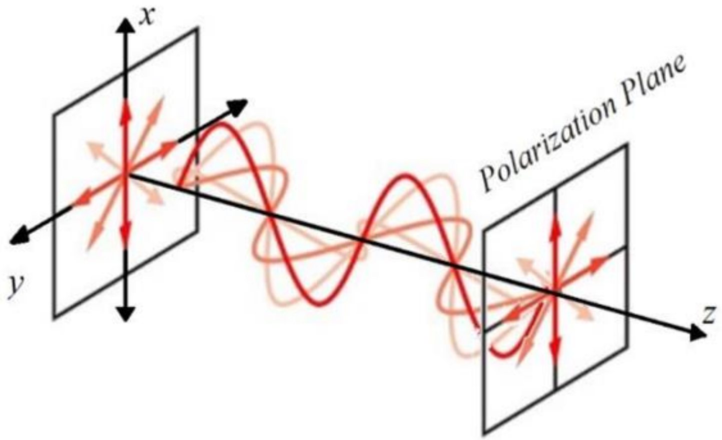
$$E = E_0 COS(\omega t - \beta z)$$
$$B = B_0 COS(\omega t - \beta z)$$

Where E_0 & B_0 are amplitude
ω is angular frequency

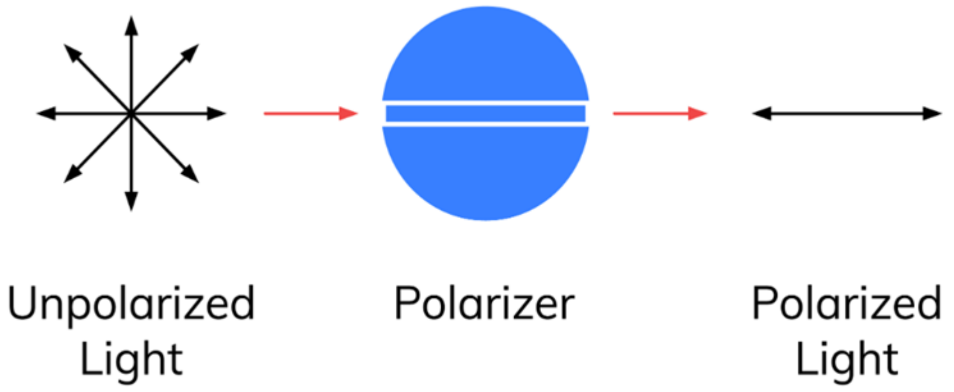
- In optical sensation magnetic field has no role,
- Hence we can say that in polarization all vibration will be of electric field vector.

The meaning of the Polarization

Polarization is a fundamental property of light . The polarization of the wave is the description of the behaviour of the vector E in the plane x, y, perpendicular to the direction of propagation z.
The plane of polarization is defined as the plane containing the propagation vector, i.e. the z axis, and the electric field vector. Consider a plane wave traveling z direction.
The electric field lies in
the x-y plane. If the direction of E changes randomly with time, the wave is said to be randomly polarized, or un-polarized



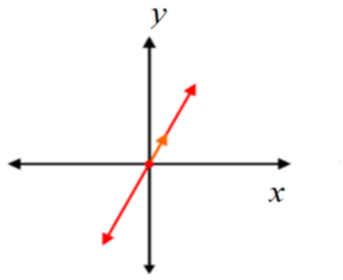
Polarization



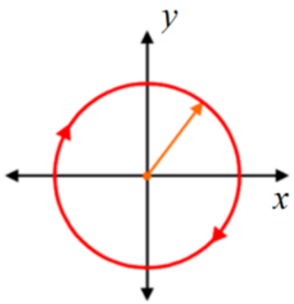
Unpolarized light is light with a random, time-varying polarization.

- Unpolarized light can be produced from the incoherent combination of vertical and horizontal linearly polarized light, or right- and left-handed circularly polarized light.
- A polarizer or polariser is an optical filter that lets light waves of a specific polarization pass through while blocking light waves of other polarizations. It can filter a beam of light of undefined or mixed polarization into a beam of well-defined polarization, known as polarized light.

- Polarization is controlled by the electric field direction in the x-y plane of the light traveling in z direction.
- There are three different types of polarization

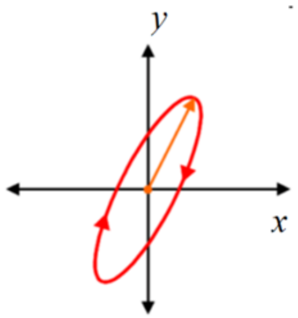


1-Linear Polarization



b

2-Circular Polarization

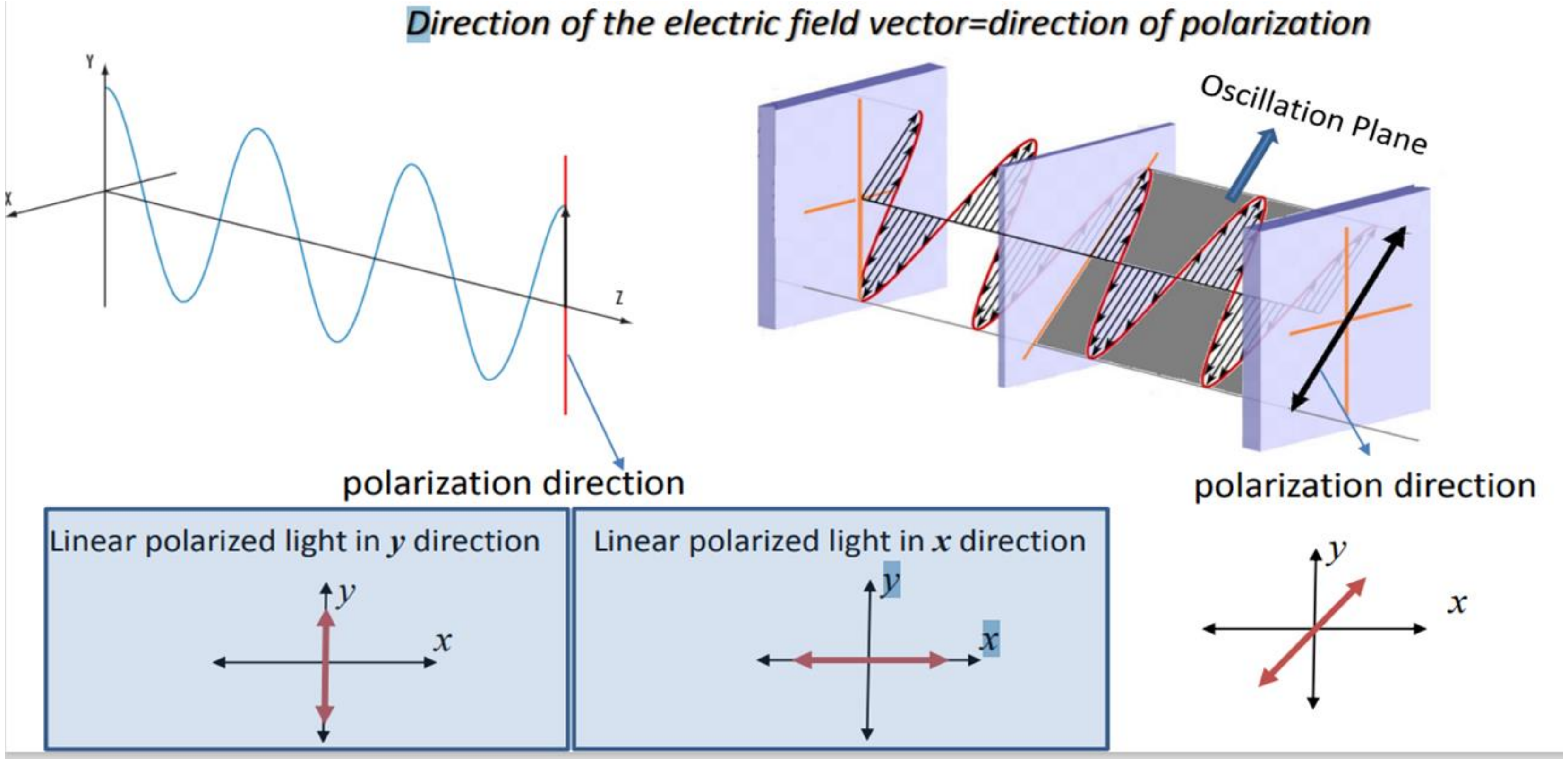


c

3-Elliptical Polarization

Linear Polarization

The electric field of light is confined to a single plane along the direction of propagation. All the electric field vectors oscillate in the same plane. They parallel to a fixed direction referred to as the polarization direction.



Linear Polarization

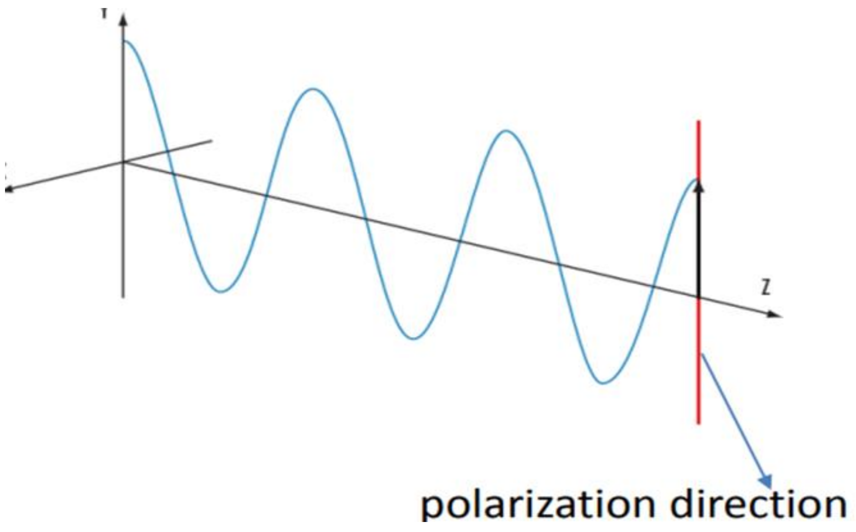
In linear polarization, the electric field (E) oscillates in a single direction or plane.

The mathematical description of a linearly polarized electromagnetic wave traveling in the z-direction can be expressed as:

$$E(z,t)=E_0\cos(kz-\omega t+\phi)\hat{x}$$

Where:

- E_0 is the amplitude of the wave.
- k is the wave number, related to the wavelength λ by $k=2\pi/\lambda$.
- ω is the angular frequency,
- t is the time.
- z is the position along the propagation direction.
- ϕ is the phase of the wave.
- $\hat{x}$ is a unit vector



Circular Polarization

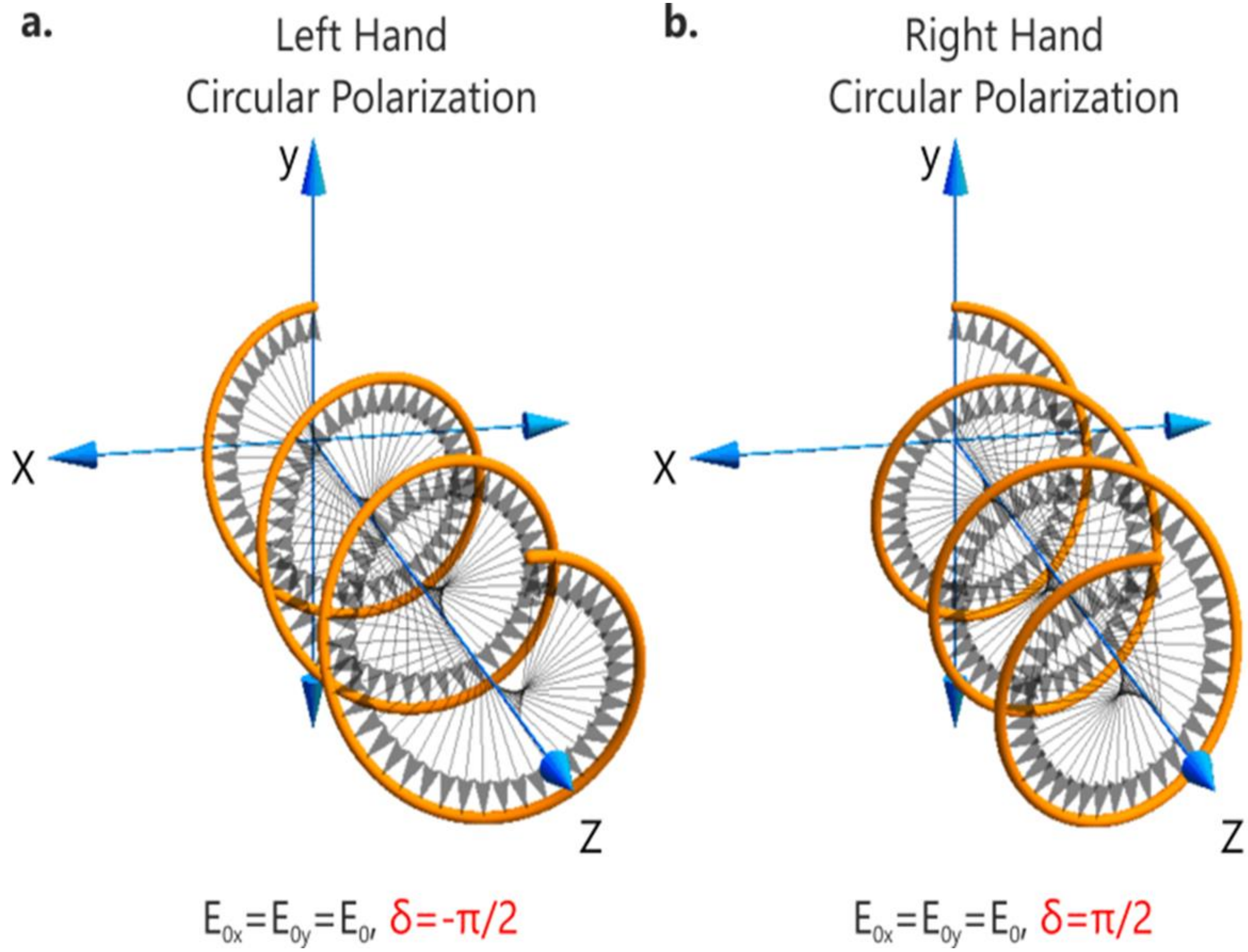
- Circular polarization occurs when two linearly polarized waves of equal amplitude and a phase difference of $\pi/2$ radians (90 degrees) combine. The electric field rotates in a circle at a constant rate as the wave propagates.
- The electric field can be represented as:

- For right-handed circular polarization,

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t)\hat{x} + E_0 \sin(kz - \omega t)\hat{y}$$

- For left-handed circular polarization

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t)\hat{x} - E_0 \sin(kz - \omega t)\hat{y}$$


a.
Left Hand
Circular Polarization
y
X
Z
$E_{0x}=E_{0y}=E_0$, $\delta=-\pi/2$
b.
Right Hand
Circular Polarization
y
X
Z
$E_{0x}=E_{0y}=E_0$, $\delta=\pi/2$

Elliptical Polarization

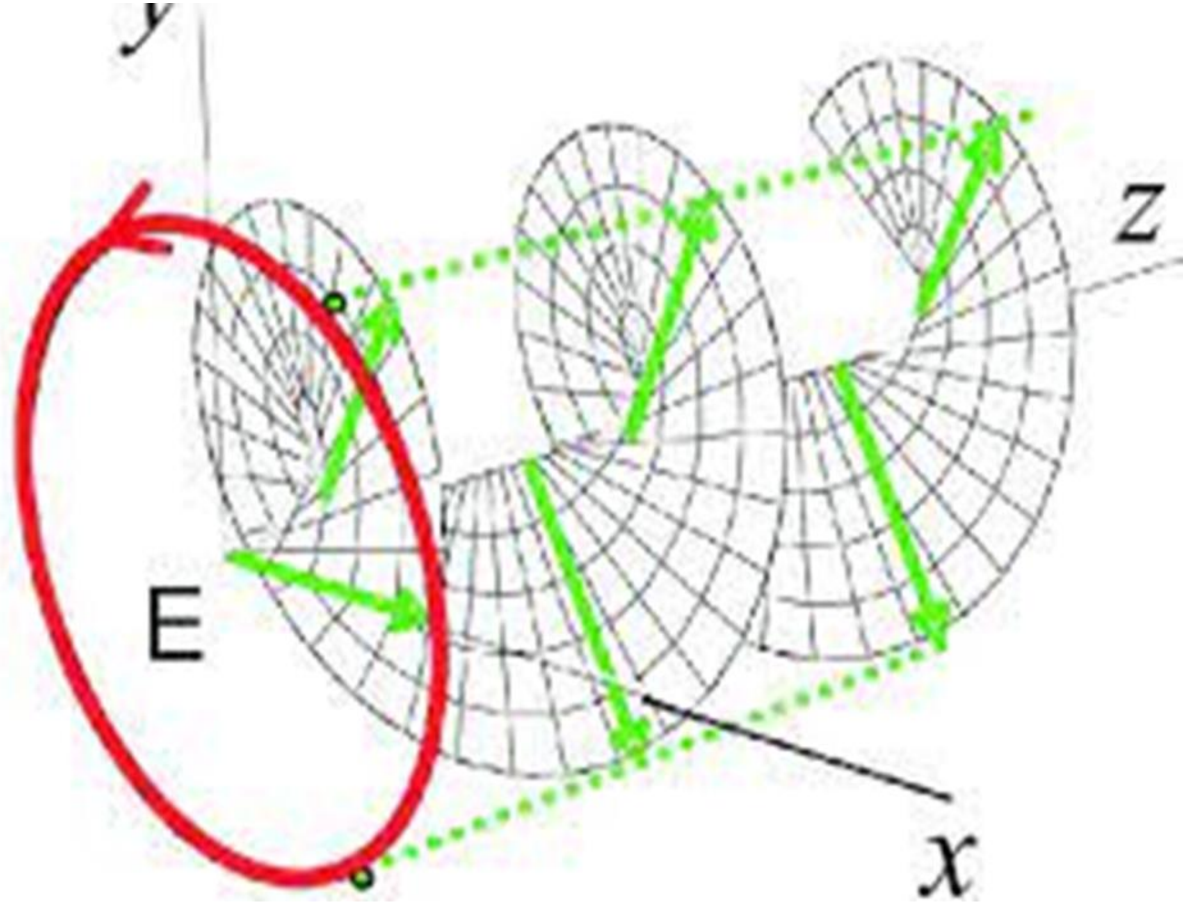
Elliptical polarization is the most general form of polarization, where the electric field describes an ellipse in any plane perpendicular to the direction of propagation. It can be seen as a superposition of two linearly polarized waves with different amplitudes and/or a phase difference other than $\pi/2$. The electric field can be represented as:

$$\mathbf{E}(z, t) = E_{0x} \cos(kz - \omega t)\hat{x} + E_{0y} \sin(kz - \omega t + \delta)\hat{y}$$

Where:

- E_{0x} and E_{0y} are the amplitudes of the electric field in the x and y directions, respectively.
- δ is the phase difference between the components of the electric field in the x and y directions.
- $\hat{x}$ and $\hat{y}$ are unit vectors in the x and y directions, respectively, orthogonal to the direction of wave propagation (z direction).

Elliptical Polarization



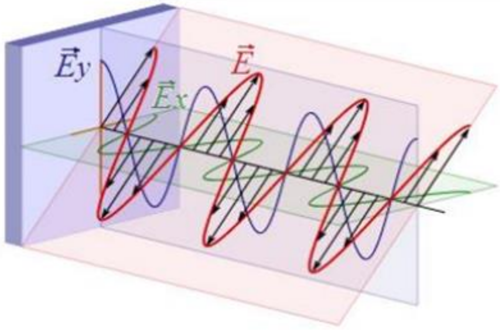
Mathematical description of polarization

$$\vec{E} = E_{ox}\cos(kz - \omega t)\hat{x} + E_{oy}\cos(kz - \omega t \pm \delta)\hat{y}$$

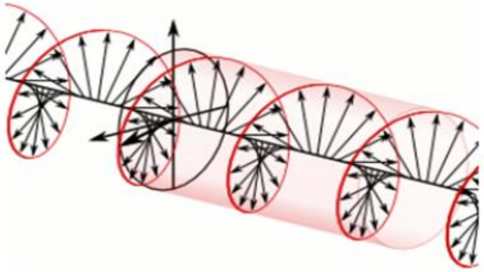
$$\delta \equiv \varphi_y - \varphi_x$$

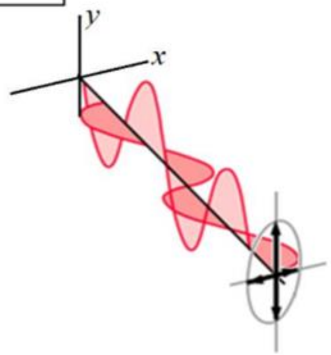
When amplitudes and phase differences are compared,

Amplitude	Phase Difference	
doesn't matter	$\delta = 0$ *or* π	***Linear Polarization***
$E_{ox} = E_{oy} = E_o$ and	$\delta = \mp\pi/2$	***Circular Polarization***
$E_{ox} \neq E_{oy}$	$\delta = \mp\pi/2$	***Elliptical Polarization***



Linear Polarization


Circular Polarization



Elliptical Polarization

POLARIZATION-LINEAR AND NONLINEAR

- When light propagates in a transparent medium (a dielectric), its electric field causes some amount of electric polarization in the medium, i.e. some density of electric dipole moment. (*This must not be confused with the polarization of the light field, which is the direction of its electric field*.)

- That polarization of the medium propagates together with the electromagnetic field in the form of a polarization wave, which has the same phase velocity as the driving field.

- Whereas at low light intensities the electric polarization is proportional to the electric field strength.

- At high intensities, the nonlinear contributions become important, as they can be produced with lasers. Such nonlinear polarization waves are essential e.g. for nonlinear processes.

POLARIZATION-LINEAR

- Linear medium: low field intensity

$D = \varepsilon_0 E + P$ where $P = \varepsilon_0 \chi \cdot E$ *i.e. P is parallel to E ,* $D = \epsilon_o E + \epsilon_o \chi E$ *or* $D = \epsilon_o E (1+ \chi)$,

$D = \varepsilon \cdot E = \varepsilon_r \varepsilon_0 E$ *i.e. D is parallel to E* $\varepsilon_r = 1 + \chi$

P= Polarization density

D= electric dipole moment/electric displacement

χ= susceptibility (degree of polarization of a dielectric material in response to an applied electric field.)

ϵ= electric permittivity of free space (electric constant=8.8541878128(13)×10^{-12} F·m^{-1});

it is a measure of how dense of an electric field is "permitted" to form in response to electric charges, and relates the units for electric charge to mechanical quantities such as length and force

E = electric field

NONLINEAR POLARIZATION

- Linear medium: low field intensity

$$D = \varepsilon_0 E + P \qquad P = \varepsilon_0 \chi \cdot E \quad \Rightarrow \quad D = \varepsilon \cdot E = \varepsilon_r \varepsilon_0 E \qquad \varepsilon_r = 1 + \chi$$

Linear polarization

- Nonlinear medium: high field intensity (electrical/optical)

$$D = \varepsilon_0 E + P \qquad P = \varepsilon_0 (\chi \cdot E + \chi^{(2)} \cdot E^2 + \chi^{(3)} \cdot E^3 + \ldots) = P_L + P_{NL}$$

$\chi^{(2)}$: 2nd order nonlinear susceptibility tensor

$\chi^{(3)}$: 3rd order nonlinear susceptibility tensor

Nonlinear polarization

R. W. Boyd, "Nonlinear optics" Elsevier (2003).

2nd order nonlinearity is absent in crystals with centro-symmetry

2^{nd} Order NONLINEAR POLARIZATION

The second (lowest) order of nonlinear polarization can arise from a $\chi^{(2)}$ **nonlinearity** which can occur only in **nonlinear crystal materials** with a **non-centrosymmetric crystal structure**. (Nonlinear effects at crystal surfaces are an exception.) The nonlinear polarization then has a component which depends quadratically on the electric field of an incident light wave. More precisely, the tensor nature of the nonlinear susceptibility needs to be considered:

$$P_i(t) = \varepsilon_0 \sum_j \chi_{ij} E_j(t) + \sum_{j,k} \varepsilon_0 \chi_{ijk}^{(2)} E_j(t) E_k(t)$$

where P_i is the i-th Cartesian coordinate of the polarization, $\chi^{(2)}$ is the nonlinear susceptibility, and $E(t)$ is the optical electric field. More commonly, this is written as

$$P_i(t) = \varepsilon_0 \sum_j \chi_{ij} E_j(t) + 2\sum_{j,k} d_{ijk} E_j(t) E_k(t)$$

with the nonlinear tensor d. Many tensor components can actually be zero for symmetry reasons, depending on the crystal class.

In general, the direction of the nonlinear polarization will not be the same as that the polarization direction of the input light

3^{rd} Order NONLINEAR POLARIZATION

- The next higher order of nonlinear polarization can arise from a $\chi^{(3)}$ nonlinearity, as it occurs in basically all media. This can give rise to various phenomena:
- The **Kerr effect** can be described as a nonlinear modification of the **refractive index**, It leads to phenomena such as **self-phase modulation** and **cross-phase modulation**, and also to **Kerr lensing** and **four-wave mixing**.
- A **delayed nonlinear response** leads to stimulated **Raman scattering** and **Brillouin scattering.**

ATOMIC ORIGIN OF OPTICAL NONLINEARITY

Induced dipole moment $P = n_e \cdot e \cdot r$ $\quad D = \varepsilon_0 E + P$

Charge density (n_e) · Electron charge (e) · Displacement (r)

- Simple harmonic oscillator model (linear)

$$m \cdot \frac{d^2 \bar{r}}{dt^2} + m\gamma \cdot \frac{d\bar{r}}{dt} + \omega_b{}^2 \bar{r} = e\bar{E}$$

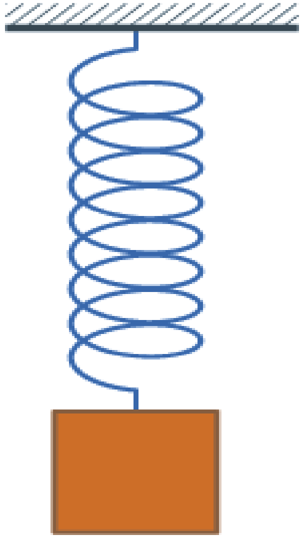
Lorentz-Drude

Damping · Restoring · Driving

Atomic nucleus

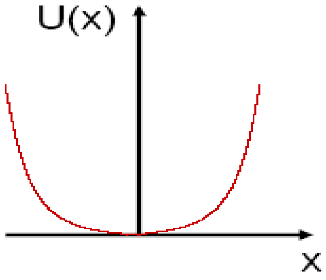
Restoring force

Electronic charge



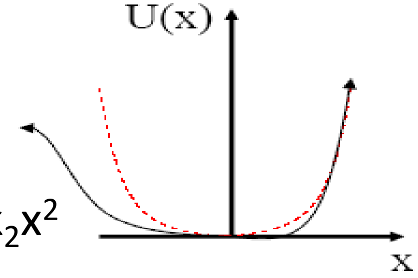
Linear polarization: parabolic potential

F = kx



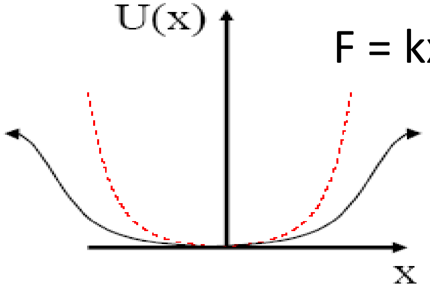
2nd order nonlinearity: Pockels media

F = kx + $k_2 x^2$



3rd order nonlinearity: Kerr media

F = kx + $k_3 x^3$



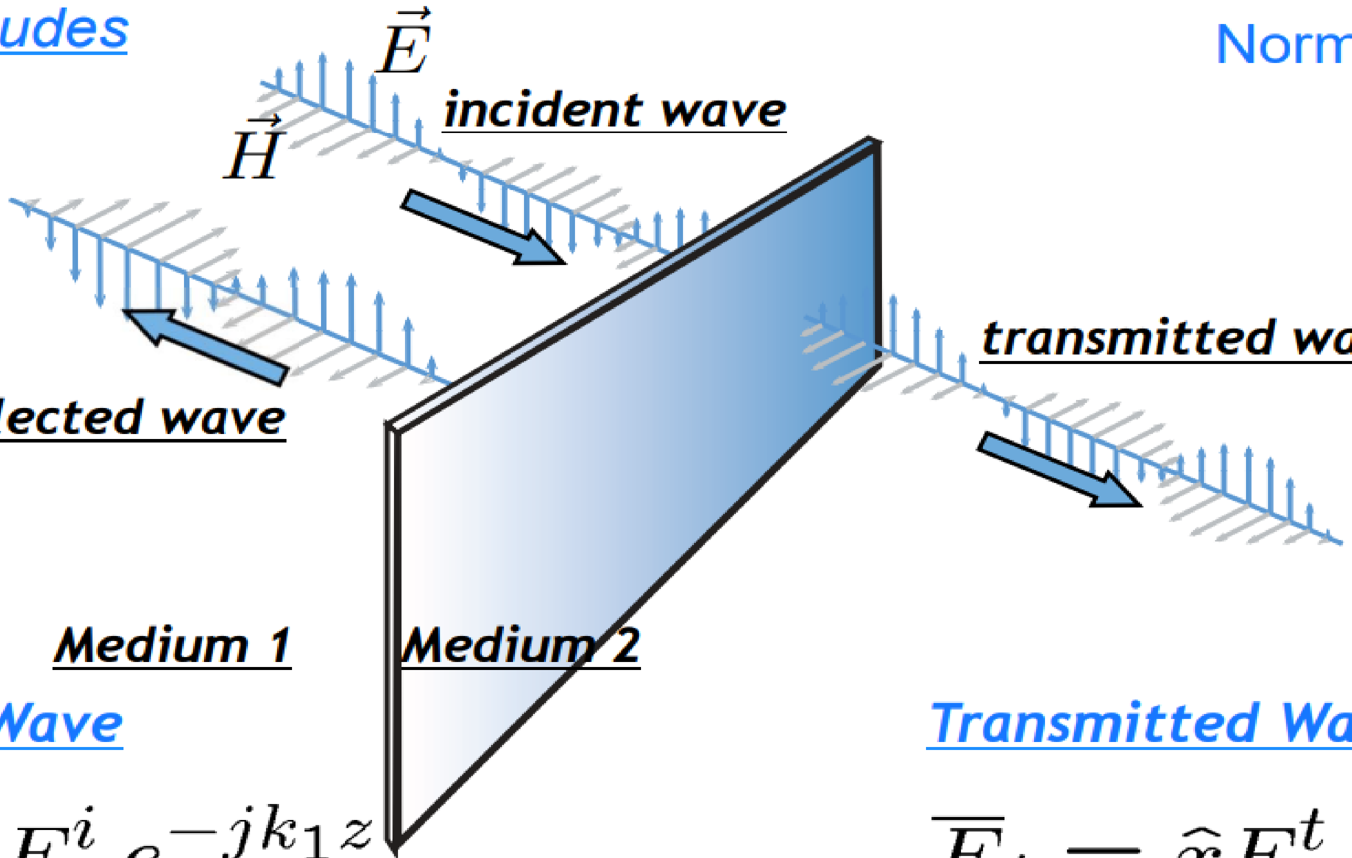
2nd order nonlinearity is absent in crystals with centro-symmetry!

G. P. Agrawal, "Nonlinear effects in Silicon Waveguides" (2007)

INCIDENT AND TRANSMITTED WAVES

same amplitudes

Normal Incidence



Incident Wave

$$\overline{E}_i = \hat{x} E_o^i e^{-jk_1 z}$$

$$k_1 = \omega\sqrt{\epsilon_1 \mu_1}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

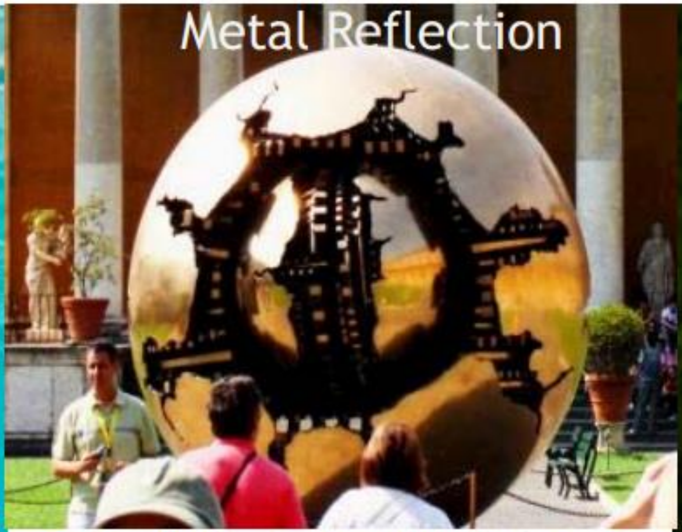
Transmitted Wave

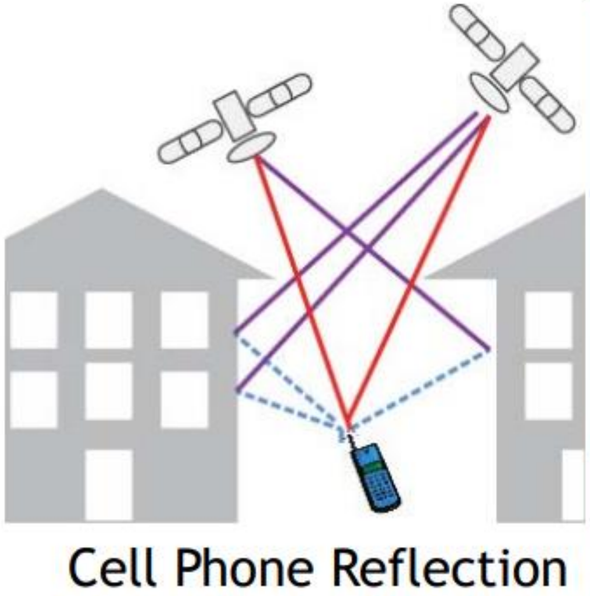
$$\overline{E}_t = \hat{x} E_o^t e^{-jk_2 z}$$

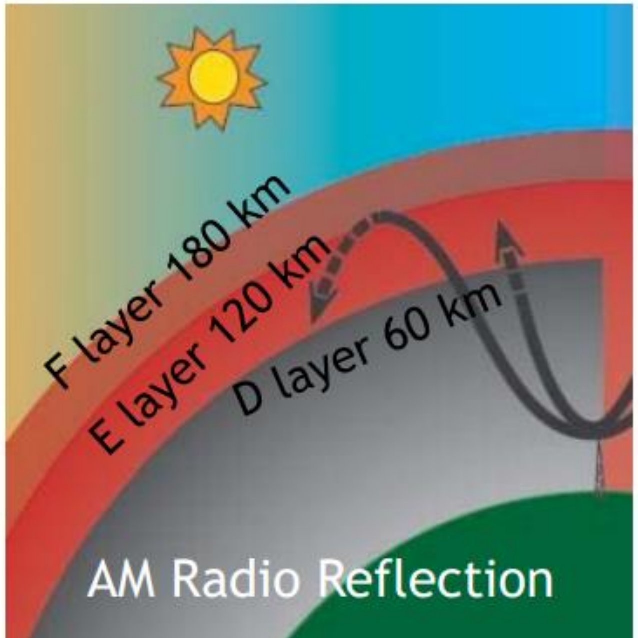
$$k_2 = \omega\sqrt{\epsilon_2 \mu_2}$$

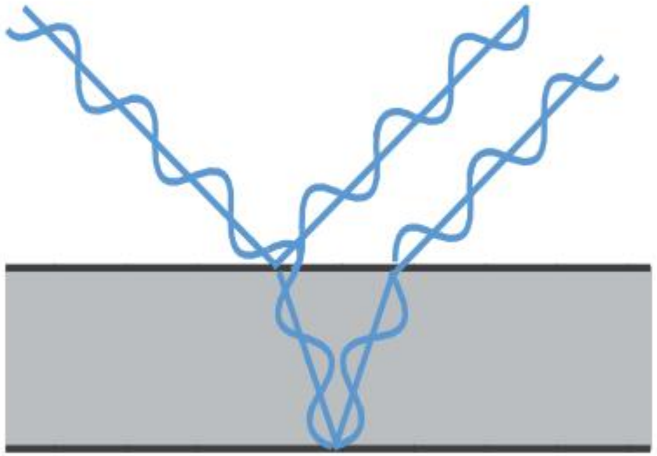
$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

EM Wave Reflection

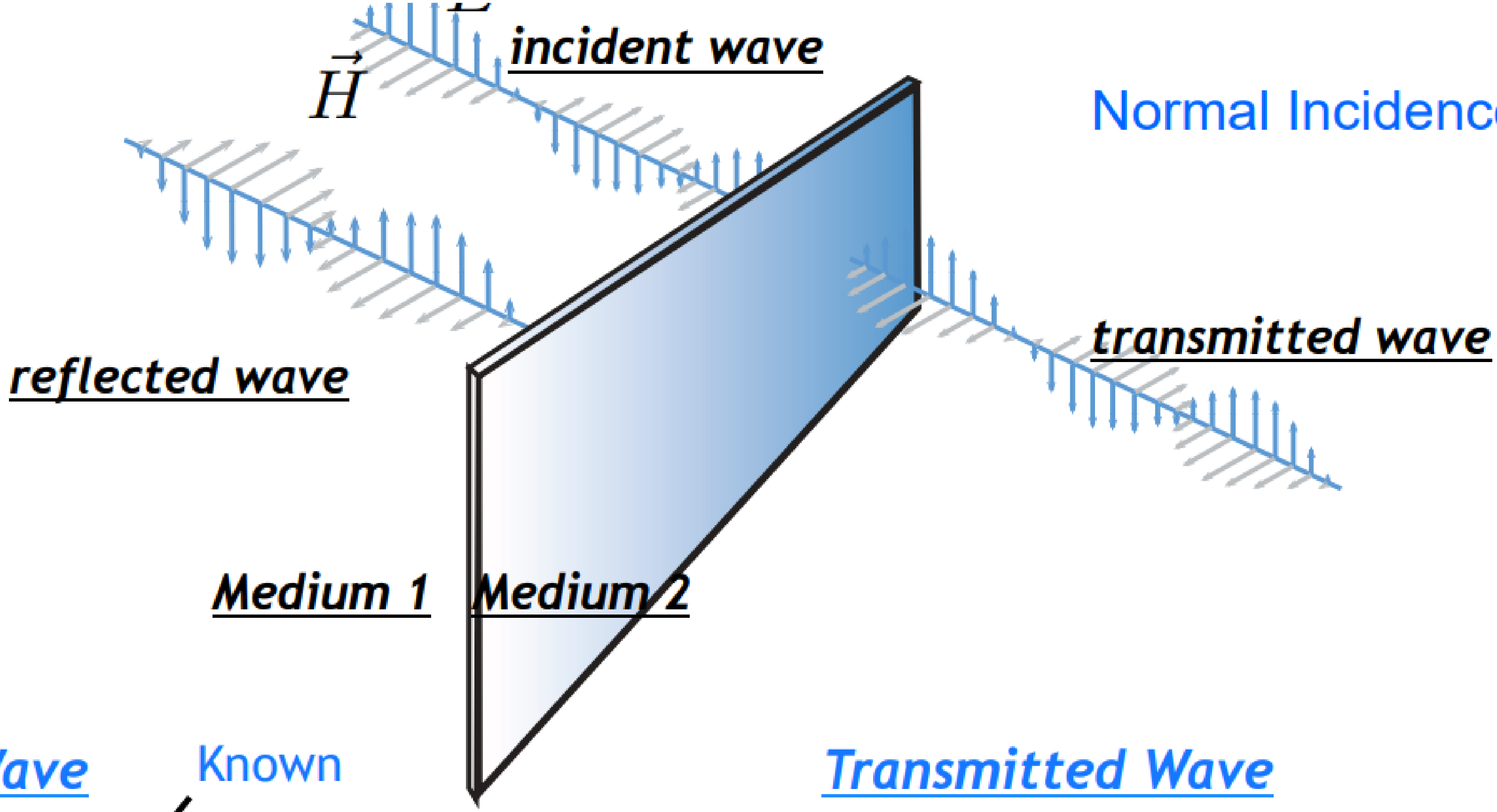








Normal Incidence



Incident Wave

Known

$$\overline{E}_i = \hat{x} E_o^i e^{-jk_1 z}$$

Reflected Wave

$$\overline{E}_r = \hat{x} E_o^r e^{+jk_1 z}$$

Transmitted Wave

$$\overline{E}_t = \hat{x} E_o^t e^{-jk_2 z}$$

Define reflection coefficient as $r = \frac{E_o^r}{E_o^i}$

Define transmission coefficient as $t = \frac{E_o^t}{E_o^i}$

- Define the ***reflection coefficient*** as

$$r = \frac{E_o^r}{E_o^i} = \frac{n_1 - n_2}{n_1 + n_2}$$

- Define the ***transmission coefficient*** as

$$t = \frac{E_o^t}{E_o^i} = \frac{2\,n_1}{n_1 + n_2}$$

Reflectivity & Transmissivity of EM Waves

- Note that $1 + r = t$
- The definitions of the reflection and transmission coefficients do generalize to the case of lossy media.
- <u>For loss-less media</u>, r and t are real:

  $-1 \leq r \leq +1 \qquad\qquad 0 \leq t \leq 2$
- <u>For lossy media</u>, r and t are complex:

  $|r| \leq 1 \qquad\qquad |t| \leq 2$
- Incident Energy = Reflected Energy + Transmitted Energy

  $R = |r|^2$ … fraction of incident power that is reflected

  $T = 1 - R$ … fraction of incident power that is transmitted

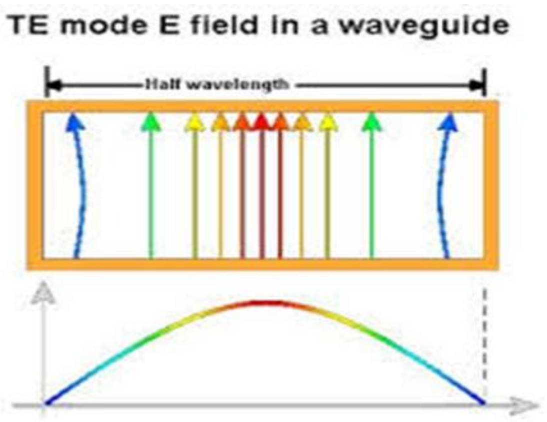
WAVEGUIDE MODE

- TE
- TM
- TEM

Waveguide Modes: TE, TM, TEM

- The mode of a waveguide defined as the transverse field pattern whose amplitude and polarization profiles remain constant along the longitudinal or z coordinate of the waveguide.

•***TE mode:*** This waveguide mode is dependent upon the transverse electric waves, also sometimes called H waves, characterised by the fact that the electric vector (E) being always perpendicular to the direction of propagation.



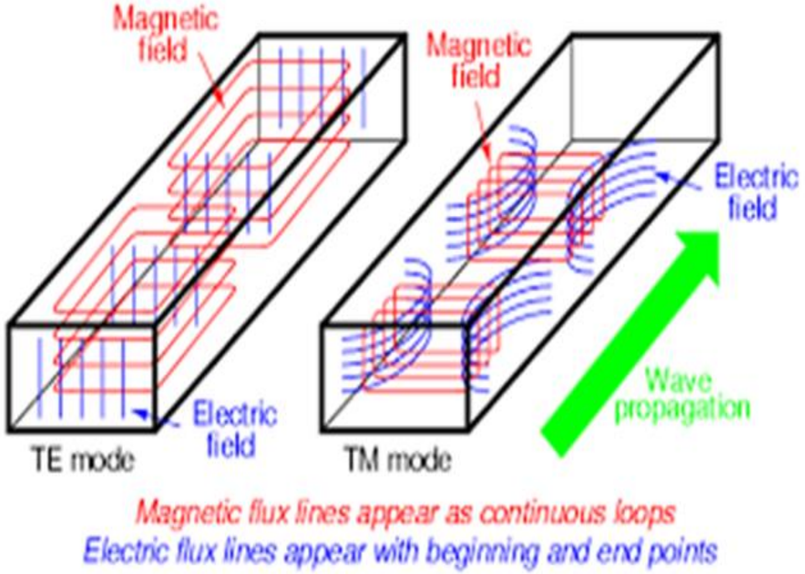
- In TE mode ,the electric field is transverse to the direction of propagation ,while the magnetic field has both transverse and longitudinal component .i.e $E_z=0, H_z \neq 0$.
- TE mode is also represented as

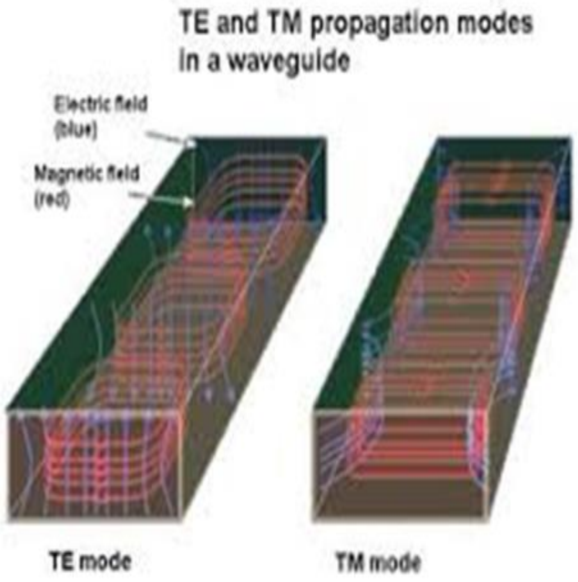
TE_{mn} ,where m=no. of half sinusoidal cycle variation along x-axis

n=no. of half sinusoidal cycle variation along y-axis.

E	H
E_X	H_X
E_Y	H_Y
$E_{Z=0}$	$H_{Z\neq0}$

•***TM mode:*** Transverse magnetic waves, also called E waves are characterized by the fact that the magnetic vector (H) is always perpendicular to the direction of propagation.





- In TM mode ,the magnetic field is transverse to the direction of propagation ,while the electric field has both transverse and longitudinal component .i.e $H_z=0, E_z \neq 0$.
- TM mode is also represented as

TM_{mn} ,where m=no. of half sinusoidal cycle variation along x-axis

n=no. of half sinusoidal cycle variation along y-axis.

E	H
E_X	H_X
E_Y	H_Y
$E_{Z\neq0}$	$H_{Z=0}$

TEM Mode: In the Transverse Electric and Magnetic (TEM) mode, both the electric field and the magnetic field (which are always perpendicular to one another in free space) are transverse to the direction of travel. it is happening in three-dimensional space.

i.e $E_z=0$,$H_z=0$

E	H
E_X	H_X
E_Y	H_Y
$E_{Z=0}$	$H_{Z=0}$