Module-1

Coulomb's Law, Electric Field Intensity and Flux Density.

Topics Covered

1. Introduction to scalars & vectors
   Vector field & Vector algebra
   Dot & Cross product
   Rectangular/Cartesian Co-ordinate System
   cylindrical & Spherical Co-ordinate system.
2. Experimental law of Coulomb.
   Electric field intensity, Field due to a Continuous Volume charge of distribution, field of a line Charge, Electric flux density.

Books Referred:-

1) W.H. Hayt and J.A. Buck, "Engineering Electromagnetics" 7th Edition, Tata Mc-Graw Hill, 2009.

2) John Krauss and Daniel A Fleisch, "Electromagnetics with applications", 5th edition - Mc-Graw-Hill.

3) Field and wave Electromagnetics - David K. Cheng 2nd edition, pearson Education.

useful formula's Required to solve problems

Differention formula

1) $\sin x = \cos x$
2) $\cos x = -\sin x$
3) $\tan x = \sec^2 x$
4) $\cot x = -\operatorname{cosec}^2 x$
5) $\sec x = \sec x \tan x$
6) $\operatorname{cosec} x = -\operatorname{cosec} x \cot x$
7) $\log_e x = \frac{1}{x}$
8) $e^x = e^x$

Product rule

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Integration

1) $\frac{1}{x} = \log_e x$
2) $e^x = e^x$
3) $\sin x = -\cos x$
4) $\cos x = \sin x$
5) $\tan x = \log(\sec x)$
6) $\cot x = \log(\sin x)$
7) $\sec x = \log(\sec x + \tan x)$
8) $\operatorname{cosec} x = \log(\operatorname{cosec} x - \cot x)$

1) for a single point straight line equation is

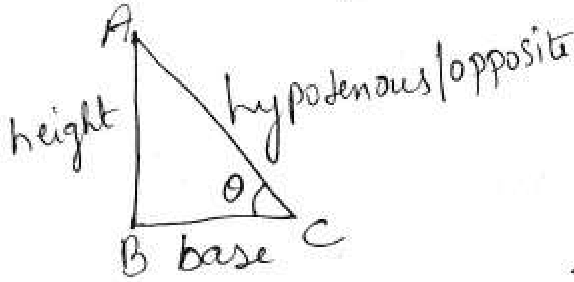
$$y = mx + c.$$

2) for two points, the straight line equation is

(x_1, y_1) (x_2, y_2)

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

right Angle triangle.



$$\sin\theta = \frac{AB}{AC},$$

$$\cos\theta = \frac{BC}{AC}$$

$$\tan\theta = \frac{AB}{BC}$$

Electromagnetics - is the study of the effects of electric charges at rest and in motion.

A field is a spatial distribution of a quantity which may or may not be a function of time. A time varying electric field is accompanied by a magnetic field and vice versa.

Time varying electric and magnetic field are coupled resulting in an electromagnetic field.

Electric Field - An electric field is a force field that acts upon material bodies by virtue of their property of charge.

Magnetic Field - A magnetic field is a force field that acts upon charges in motion.

Principle is same as that of a gravitational field.

Electromagnetics is important because it provides a real-World, 3-D understanding of electricity and magnetism.

Electromagnetic theory is necessary in understanding the principle of atoms, radar, satellite communication, microwave devices, optical fiber communication etc.

Application of EM

1) Medical field like laser therapy, X-ray.

3) Bar Code Reader & CD players.
4) Relay in an electrical device use EM fields to engage or disengage the two different States.
5) Meters and Motors.
6) The transformer.
7) Speaker, Super Conductors etc.

Vector Analysis -

Vector analysis is a mathematical shorthand which greatly facilitates the analysis of electric and magnetic fields.

Scalar -

A Scalar is a quantity which has only magnitude, represented by a single real number.
Eg:- Charge, current, energy, temperature, mass etc.

Vector -

A Vector is a quantity which has both magnitude and direction.
Eg:- electric field intensity, magnetic field intensity, force Velocity, displacement etc.

Scalar field -

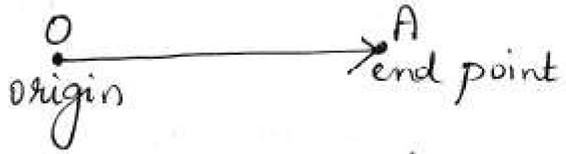
A field is a region in which a particular physical function has a value at each and every point in that region. The distribution of a Scalar quantity with a definite position in a space is called scalar field. Eg:- atmospheric temperature

Vector field.

If a quantity which is specified in a region to define a field is a vector then the corresponding field is called a Vector field.

Eg:- the gravitational force on a mass in a space.

Representation of a Vector



$= \vec{OA}$

In 2-D a vector can be represented by a straight line with an arrow in a plane. The length of the segment is the magnitude of a vector and arrow indicates the direction, in a given Co-ordinate System.

Vector Algebra

The addition of vectors follows the parallelogram law.

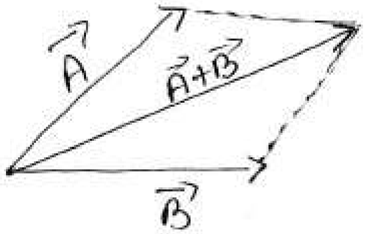


fig shows the sum of two vectors $\vec{A}$ & $\vec{B}$.

$\vec{A}+\vec{B} = \vec{B}+\vec{A}$ $\Rightarrow$ vector addition obeys Cumulative law and also obeys associative law.

$$\vec{A} + (\vec{B}+\vec{C}) = (\vec{A}+\vec{B}) + \vec{C}$$

Co-planar vectors are vectors lying in a Common plane.

The rule for the subtraction of vectors follows easily from that for addition, $\vec{A}-\vec{B}$ or $\vec{A}+(-\vec{B})$ [the sign or direction of 2^{nd} vector is reversed and then added to the 1^{st} vector.]

Vectors may be multiplied by scalars. The magnitude of the vector changes, but its direction does not when the scalar is positive, although it reverses direction when multiplied by a negative scalar.

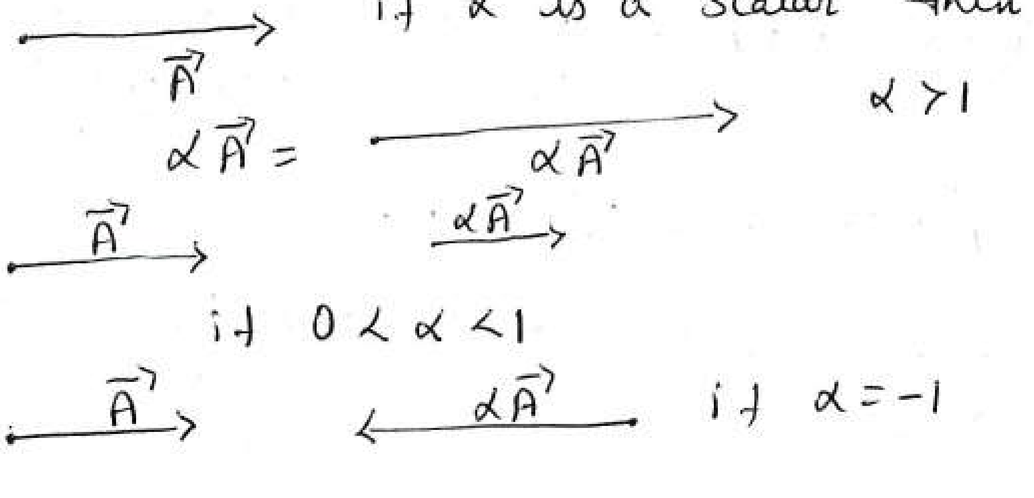
Multiplication of a vector by a scalar also obeys the associative & distributive laws.

$$(r+s)(\vec{A}+\vec{B}) = r(\vec{A}+\vec{B}) + s(\vec{A}+\vec{B})$$
$$= r\vec{A} + r\vec{B} + s\vec{A} + s\vec{B}.$$

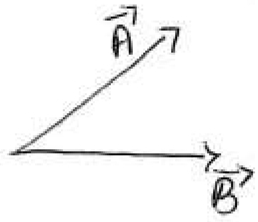
Division of a vector by a scalar is merely multiplication by the reciprocal of that scalar.

identical vectors:- two vectors are said to be equal if their difference is zero or $\vec{A}-\vec{B}=0$ or $\vec{A}=\vec{B}$.

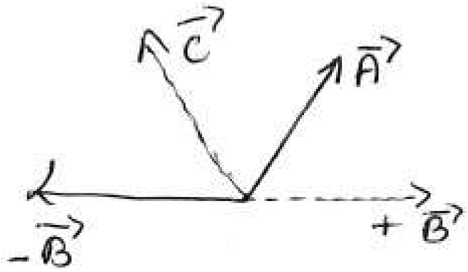
multiplication of vectors:



Subtraction of Vectors



$\vec{C} = \vec{A} - \vec{B} \Rightarrow$



Co-ordinate system

To describe a vector accurately some specific length, directions, angles, projections or components must be given.

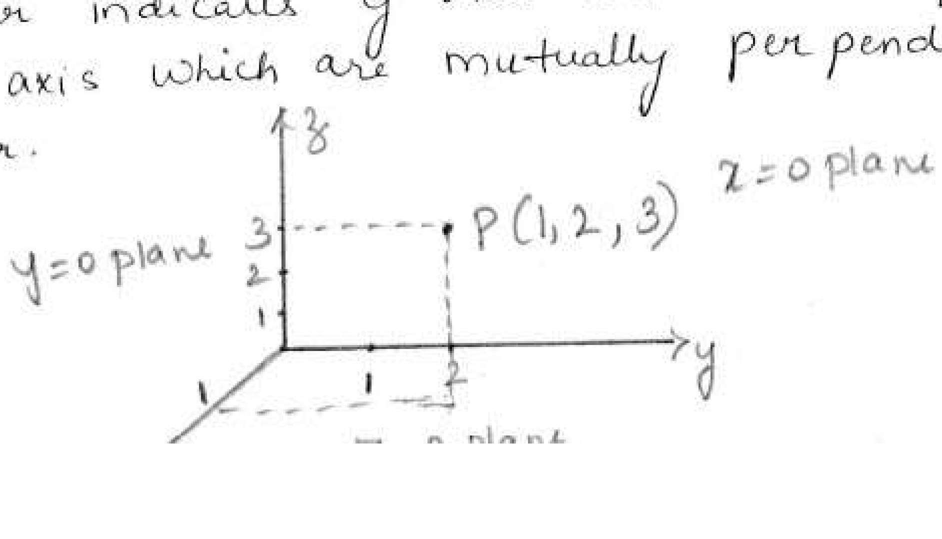
The most important co-ordinate system available are :-

1) Cartesian or rectangular co-ordinate system.
2) cylindrical co-ordinate system
3) Spherical co-ordinate system.

1) Cartesian / rectangular Co-ordinate system:

This system has three co-ordinate axis represented as x, y and z which are mutually at right angles to each other. These 3 axes intersect at a common point called origin of the system.

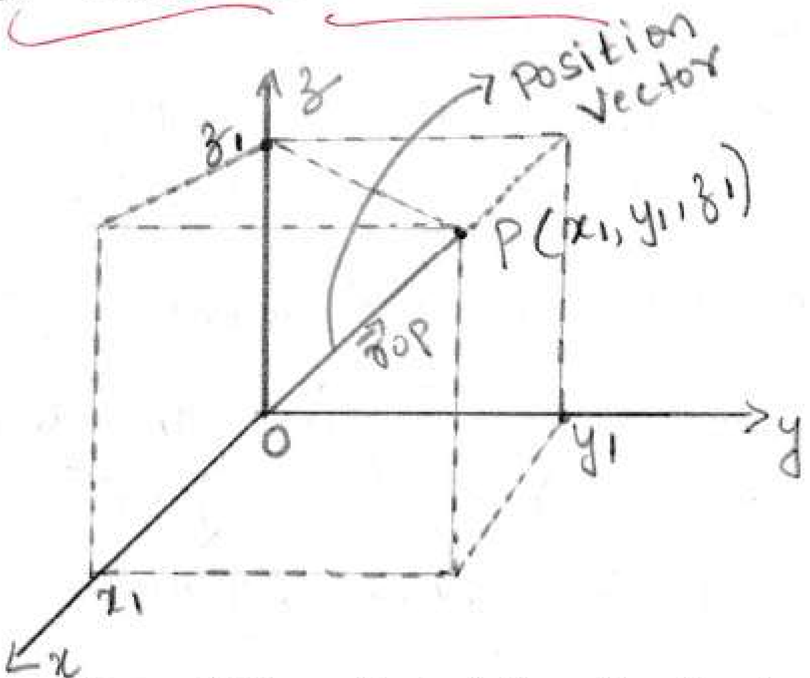
This is a system right handed co-ordinate system. If right hand is used then thumb indicates x-axis, the fore finger indicates y axis and middle finger indicates z axis which are mutually perpendicular to each other.



A point 'P' is located by giving its x, y, z Co-ordinate. These are distance from the origin to the intersection of a ⊥r dropped from the point to the x, y, z axis.

$[-\infty \leq x, y, z \leq \infty]$.

Position and distance vectors



Consider a point $P(x_1, y_1, z_1)$ in Cartesian Co-ordinate system. Then the position vector of point 'P' is represented by the distance of point 'P' from the origin, directed from origin to point 'P'. this is called 'radius/position' vector.

The three components of this position vector $\vec{r}_{op}$ are three vectors oriented along the three-coordinate axes with the magnitudes x_1, y_1 and z_1. Thus the position vector of point 'P' can be represented as,

$$\vec{r}_{op} = x_1\hat{a}_x + y_1\hat{a}_y + z_1\hat{a}_z$$

[Note: $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ are unit vectors]

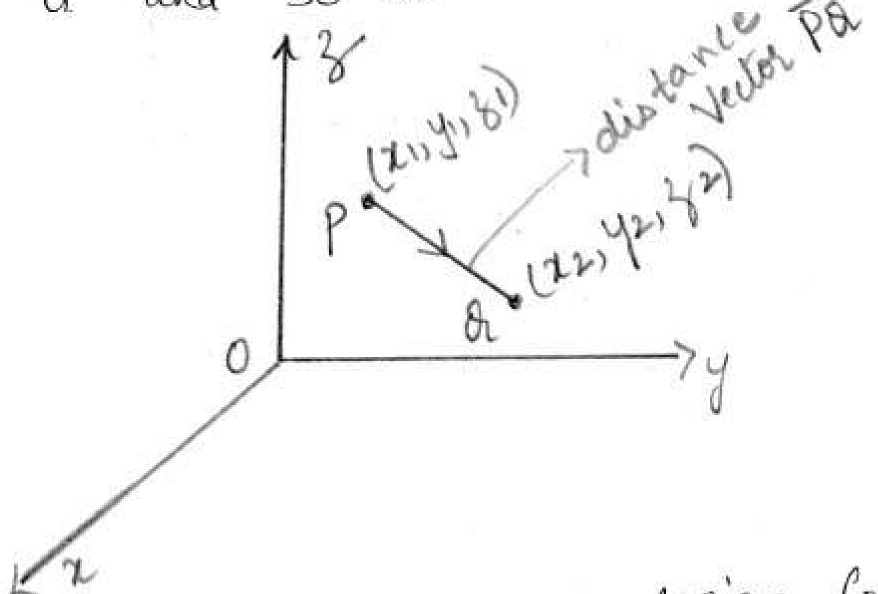
$$\vec{r}_{op} = x_1\hat{a}_x + y_1\hat{a}_y + z_1\hat{a}_z$$

the magnitude of this vector interms of three mutually perpendicular components is given by, If point has co-ordinates (1, 2, 3) then the position vector is,

$$\vec{r}_{op} = 1\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$$

ie $|\vec{r}_{op}| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{14} = \underline{\underline{3.7416}}$

Many a times the position vector is denoted by the vector representing that point itself i.e, for point P the position vector is $\vec{P}$ and point Q it is $\vec{Q}$ and so on.



Consider the two points in a cartesian co-ordinate system P and Q with the co-ordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.

The individual position vectors of the points are,

$$\vec{P} = x_1\hat{a}_x + y_1\hat{a}_y + z_1\hat{a}_z$$
$$\vec{Q} = x_2\hat{a}_x + y_2\hat{a}_y + z_2\hat{a}_z$$

then the distance or the displacement from P to Q is represented by a distance vector $\vec{PQ}$ ie is given by

$$\vec{PQ} = \vec{Q} - \vec{P} = [x_2 - x_1]\hat{a}_x + [y_2 - y_1]\hat{a}_y + [z_2 - z_1]\hat{a}_z$$

the magnitude of this vector is given by, If a vector has

$$\vec{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$\therefore$ A unit vector is given by

$$\vec{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{\vec{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

1) Obtain the unit vector in the direction from the origin towards the point P (3, −3, −2).

Soln:- the Origin O (0, 0, 0) has Co-ordinate points while P (3, −3, −2) hence the distance vector $\vec{OP}$ is.

$$\vec{OP} = (3-0)\hat{a}_x + (-3-0)\hat{a}_y + (-2-0)\hat{a}_z$$

$$= 3\hat{a}_x - 3\hat{a}_y - 2\hat{a}_z$$

$$|\vec{OP}| = \sqrt{(3)^2 + (-3)^2 + (-2)^2} = 4.6904$$

Hence unit vector along the direction $\vec{OP}$ is

$$\hat{a}_{op} = \frac{\vec{OP}}{|\vec{OP}|} = \frac{3\hat{a}_x - 3\hat{a}_y - 2\hat{a}_z}{4.6904}$$

$$= 0.6396\hat{a}_x - 0.6396\hat{a}_y - 0.4264\hat{a}_z$$

2) two points A (2, 2, 1) and B (3, −4, 2) are given in the cartesian system. Obtain the vector from A to B and a unit vector directed from A to B.

Soln:- A - Starting point, B - terminating point.

$$\vec{A} = 2\hat{a}_x + 2\hat{a}_y + \hat{a}_z, \quad \vec{B} = 3\hat{a}_x - 4\hat{a}_y + 2\hat{a}_z$$

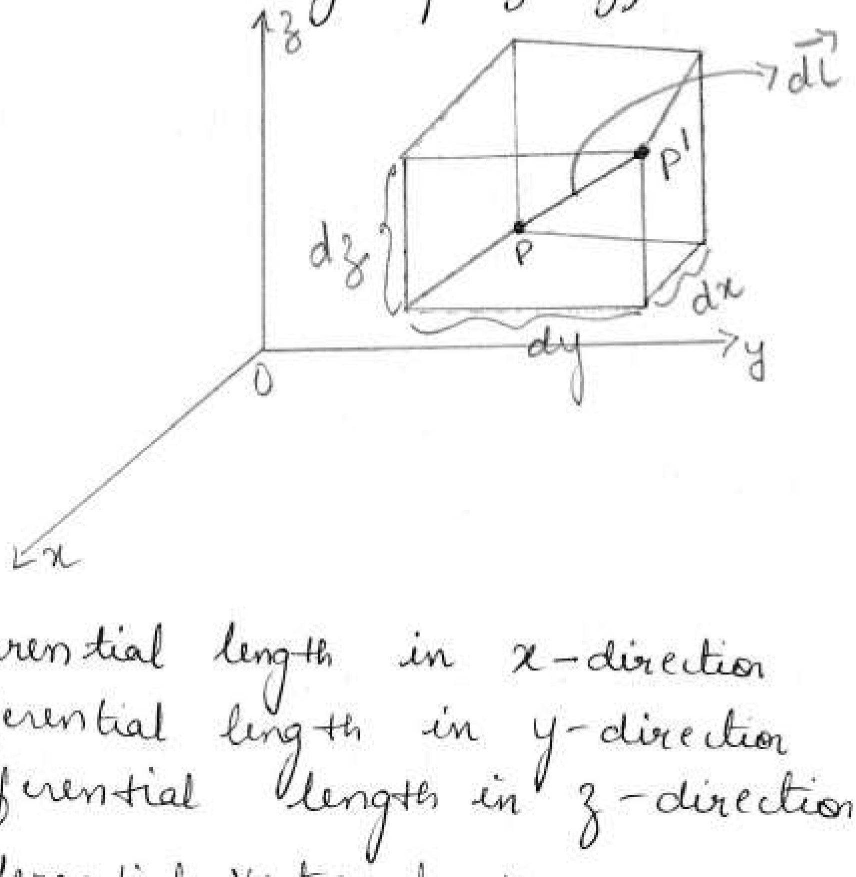
$$\vec{AB} = \vec{B} - \vec{A} = (3-2)\hat{a}_x + (-4-2)\hat{a}_y + (2-1)\hat{a}_z$$

$$|\overrightarrow{AB}| = \sqrt{(1)^2 + (-6)^2 + (1)^2} = 6.1644$$

$$\hat{a}_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\hat{a}_x - 6\hat{a}_y + \hat{a}_z}{6.1644} = 0.1622\hat{a}_x - 0.9733\hat{a}_y + 0.1622\hat{a}_z$$

Differential elements in Cartesian Co-ordinate System

Consider a point $P(x, y, z)$ in the rectangular Co-ordinate System. Let us increase each Co-ordinate by a differential amount. A new point 'p' will be Obtained having Co-ordinates $(x+dx,\ y+dy,\ z+dz)$.



dx = differential length in x-direction
dy = differential length in y-direction
dz = differential length in z-direction

∴ differential Vector length

$$\boxed{\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z}$$

this is the Vector joining point 'P' to new point 'P''.

$P \rightarrow$ is the intersection of 3 planes.

from Original three planes.

The 6 planes together define a differential volume which is a rectangular parallelepiped, the diagonal of this Parallelepiped is the differential vector length.

Magnitude of this $\vec{dl}$ is,

$$|\vec{dL}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

Hence the differential Volume is

$$\boxed{dV = dx\,dy\,dz}$$

the differential surface areas are:-

$$\vec{dS_x} = dy\,dz\,\hat{a}_x$$

$$\vec{dS_y} = dz\,dx\,\hat{a}_y$$

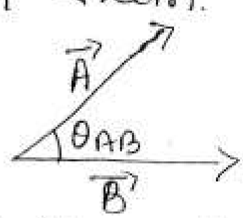
$$\vec{dS_z} = dx\,dy\,\hat{a}_z$$

Vector Multiplication:

DOT Product

Given two vectors A and B, the dot product or Scalar product is defined as the product of the magnitude of A and the magnitude of B and the Cosine of the Smaller angle between them.

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta_{AB}}$$



The dot product obeys Cummulative law,

$$\boxed{\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}}$$

[Note:- $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB}$ → the result of such a dot product is scalar hence it is also called scalar product.]

dot product obeys distributive law,

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

application of dot product

A constant force F applied over a straight displacement L does an amount of work ($FL\cos\theta$) which can be written as $F\cdot L$ or $\text{work} = \int \vec{F}\cdot d\vec{L}$

[$\int$ integration → is necessary to find the total work].

Consider two vectors whose rectangular components are given, such as

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

& $$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$$

$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

∵ becoz $\hat{a}_x\cdot\hat{a}_y = \hat{a}_y\cdot\hat{a}_x = \hat{a}_x\cdot\hat{a}_z = \hat{a}_z\cdot\hat{a}_x = \hat{a}_y\cdot\hat{a}_z = \hat{a}_z\cdot\hat{a}_y = 0$

$$\hat{a}_x\cdot\hat{a}_x = \hat{a}_y\cdot\hat{a}_y = \hat{a}_z\cdot\hat{a}_z = 1$$

Since the angle between two different unit vectors of the rectangular co-ordinate system is 90°.

A vector dotted with itself yields the magnitude squared.

$$\vec{A}\cdot\vec{A} = (\vec{A})^2 = |\vec{A}|^2$$

If two vectors are $\perp r$ to each other i.e, $\theta = 90°$.
then $\cos\theta_{AB} = 0$ thus

$$\boxed{\vec{A}\cdot\vec{B} = 0}$$

application of Dot products are:-

(a) To determine the angle between the two vectors

$$\boxed{\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}\right]}$$

(b) to find the Component of a vector in a given direction.

Consider a vector $\vec{P}$ and a unit vector $\hat{a}$.

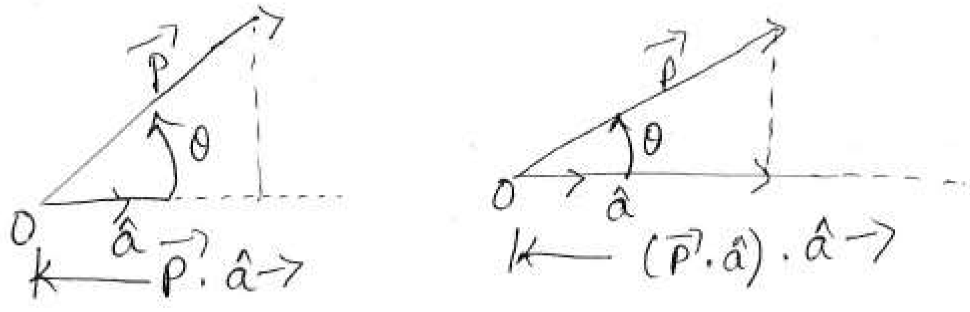


fig (a)

the Component of vector $\vec{P}$ in the direction of unit vector $\hat{a}$ is $\vec{P}\cdot\hat{a}$. This is a scalar quantity.

$$\boxed{\vec{P}\cdot\hat{a} = |\vec{P}|\,|\hat{a}|\cos\theta_{Pa}}$$

Ec Component vector $= (\vec{P}\cdot\hat{a})\,\hat{a}$

$\vec{P}\cdot\hat{a}$ is the projection of $\vec{P}$ in the $\hat{a}$ direction.

1) given the two vectors.

$\vec{A} = 2\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z$ & $\vec{B} = 3\hat{a}_x + 5\hat{a}_y + 2\hat{a}_z$

find the dot product & the angle b/w the two Vectors.

Soln:- dot product is

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$= (2\times 3) + (-5)(5) + (-4)(2)$$
$$= 6 - 25 - 8 = -27$$

2) given a vector field $\vec{G} = y\hat{a}_x - 2.5x\,\hat{a}_y + 3\hat{a}_z$ and point $Q(5,4,1)$. So find

(a) $\vec{G}$ at Q.

(b) the scalar component of $\vec{G}$ at Q in the direction of $\hat{a}_N = \frac{1}{3}(2\hat{a}_x + \hat{a}_y - 2\hat{a}_z)$

(c) the vector component of G at Q in the direction of $\hat{a}_N$.

(d) angle θ_{Ga} between $\vec{G}(r_a)$ & $\hat{a}_N$

Soln:- Substituting the co-ordinates of point Q into the expression for G.

(a) $\vec{G}(r_a) = 5\hat{a}_x - 10\hat{a}_y + 3\hat{a}_z$

(b) using dot product.

$$\vec{G}\cdot\hat{a}_N = (5\hat{a}_x - 10\hat{a}_y + 3\hat{a}_z)\cdot\frac{1}{3}(2\hat{a}_x + \hat{a}_y - 2\hat{a}_z)$$
$$= \frac{1}{3}(10 - 10 - 6) = -2$$

(c) the vector Component is obtained by multiplying the scalar Component by the unit vector in the direction of $\hat{a}_N$.

$$(\vec{G}\cdot \hat{a}_N)\, \hat{a}_N = (-2)\left(-\frac{1}{3}\right)(2\hat{a}_x + \hat{a}_y - 2\hat{a}_z)$$

$$= -1.33\,\hat{a}_x - 0.667\,\hat{a}_y + 1.33\,\hat{a}_z$$

(d) the angle b/w $\vec{G}$ (r_a) and $\hat{a}_N$ is

$$\vec{G}\cdot\hat{a}_N = |G| \cos\theta_{Ga}$$

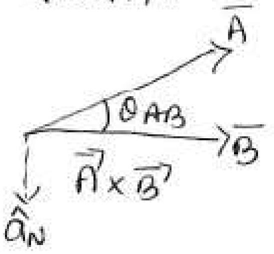
$$-2 = \sqrt{25+100+9}\, \cos\theta_{Ga}$$

$$\theta_{Ga} = \cos^{-1} \frac{-2}{\sqrt{134}} = 99.9^\circ$$

(take - sign also)

Cross Product

cross product is defined as the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and sine of the smaller angle between them.



$$\boxed{\vec{A}\times\vec{B} = |\vec{A}||\vec{B}| \sin\theta_{AB}\, \hat{a}_N}$$

$\hat{a}_N$ = unit Normal vector $\perp$r to the plane $\vec{A}$ & $\vec{B}$.

The direction of $\vec{A}\times\vec{B}$ is $\perp$r to the plane Containing $\vec{A}$ & $\vec{B}$ and is along that one of the two possible $\perp$r which is in the direction of advance of a rt-handed screw as $\vec{A}$ is turned into $\vec{B}$.

Eg:- $\vec{F} = I\vec{l}\times\vec{B} = IlB\sin\theta\,\hat{n}$

Note:-

1) Commutative law is not applicable to the cross Product.

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

2) Cross product is not associative, thus

$$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$

3) With respect to addition, the cross product is distributive thus,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

4) If two vectors are parallel i.e, in the same direction then $\theta = 0^\circ$ hence cross product of such two vectors is zero.

$$\vec{A} \times \vec{A} = 0$$

5) Cross product of unit vectors - Consider a unit vector $\hat{a}_x$, $\hat{a}_y$ & $\hat{a}_z$ which are mutually $\perp$r to each other, then

$$\hat{a}_x \times \hat{a}_y = |\hat{a}_x| \, |\hat{a}_y| \sin 90^\circ \, \hat{a}_N$$

$$\hat{a}_N = \hat{a}_z$$

$$\& \quad |\hat{a}_x| = |\hat{a}_y| = \sin(90^\circ) = 1$$

$$\therefore \quad \hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\hat{a}_y \times \hat{a}_z = \hat{a}_x$$

$$\hat{a}_z \times \hat{a}_x = \hat{a}_y$$

$$\left.\begin{aligned} \hat{a}_y \times \hat{a}_x &= -\hat{a}_z \\ \hat{a}_z \times \hat{a}_y &= -\hat{a}_x \\ \hat{a}_x \times \hat{a}_z &= -\hat{a}_y \end{aligned}\right\} \text{ If the order of unit vector is reversed.}$$

While as cross product of vector with itself is zero we can write,

$$\hat{a}_x \times \hat{a}_x = \hat{a}_y \times \hat{a}_y = \hat{a}_z \times \hat{a}_z = 0.$$

Consider two vectors in the Cartesian form as,

$$\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$$

$$\vec{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$$

then, the cross product is

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\therefore \quad = (A_y B_z - A_z B_y)\hat{a}_x - (A_x B_z - A_z B_x)\hat{a}_y + (A_x B_y - B_x A_y)\hat{a}_z$$

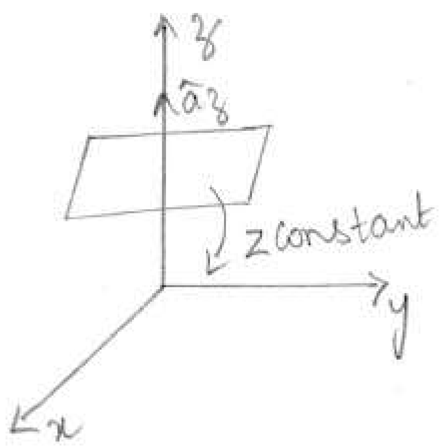
2) Cylindrical Co-ordinate System

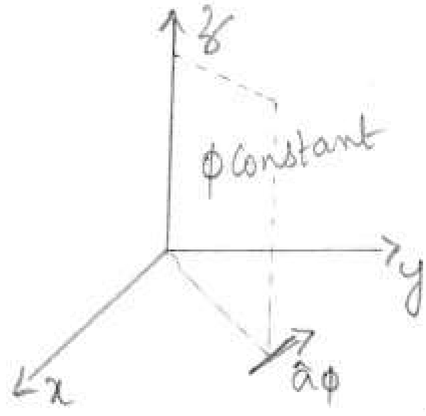
The circular cylindrical co-ordinate system is the 3D version of polar co-ordinate system. The surfaces used to define the cylindrical co-ordinate system are:-

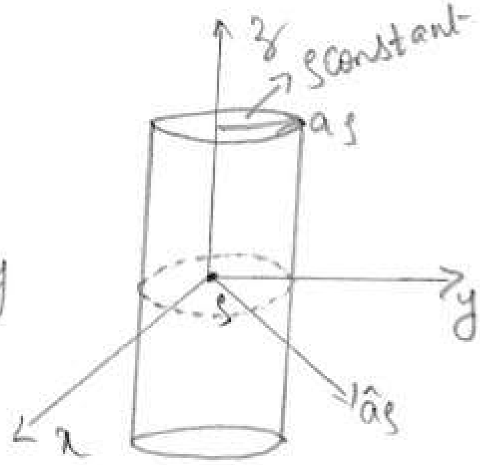
(a) Plane of constant Z which is parallel to xy plane.

(b) A cylinder of radius 'r' with z-axis as the axis of the cylinder.

(c) A half plane ⊥ to to XY plane and at an angle ϕ with respect to xz plane.







The Point 'p' in cylindrical co-cordinate system has three co-ordinates ρ, ϕ and Z.

Radius - ρ, angle - ϕ, height - Z

the ranges of the variables are,

$$0 \leq \rho \leq \infty$$

$$0 \leq \phi \leq 360^\circ \,(2\pi)$$

$$-\infty < Z \leq \infty$$

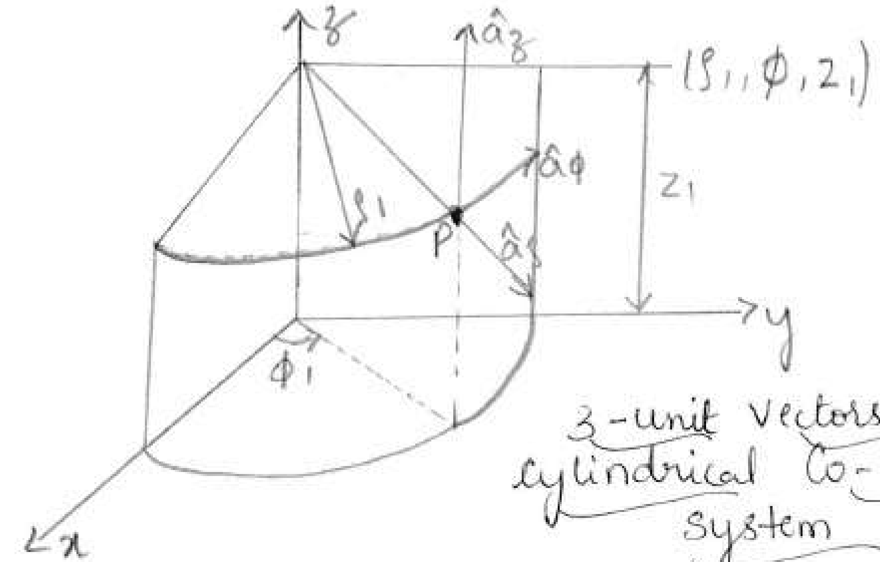
The unit vectors of cylindrical co-ordinates are $\hat{a}_\rho$, $\hat{a}_\phi$ and $\hat{a}_Z$.

The unit vector $\hat{a}_\rho$ at a point $P(\rho, \phi, Z)$ is directed radially outward, normal to the cylindrical surface at $\rho = \rho_1$. It lies in the plane $\phi = \phi_1$ and $Z = Z_1$.

The unit vector $\hat{a}_\phi$ is normal to the plane $\phi = \phi_1$. Points in the direction of increasing ϕ, lies in the plane $Z = Z_1$ and is tangent to the cylindrical surface $\rho = \rho_1$.

The unit vector $\hat{a}_Z$ is same as the unit vector $\hat{a}_Z$ of the rectangular co-ordinate system.

A differential volume element in cylindrical co-ordinates may be obtained by increasing ρ, ϕ and z by the differential elements $d\rho$, $d\phi$ and dz.



3-unit vectors of cylindrical Co-ordinate system

the surfaces

$\vec{ds_\rho} = \rho\, d\phi\, dz\, \hat{a}_\rho \rightarrow \rho$ Constant

$\vec{ds_\phi} = d\rho\, dz\, \hat{a}_\phi \rightarrow \phi$ Constant

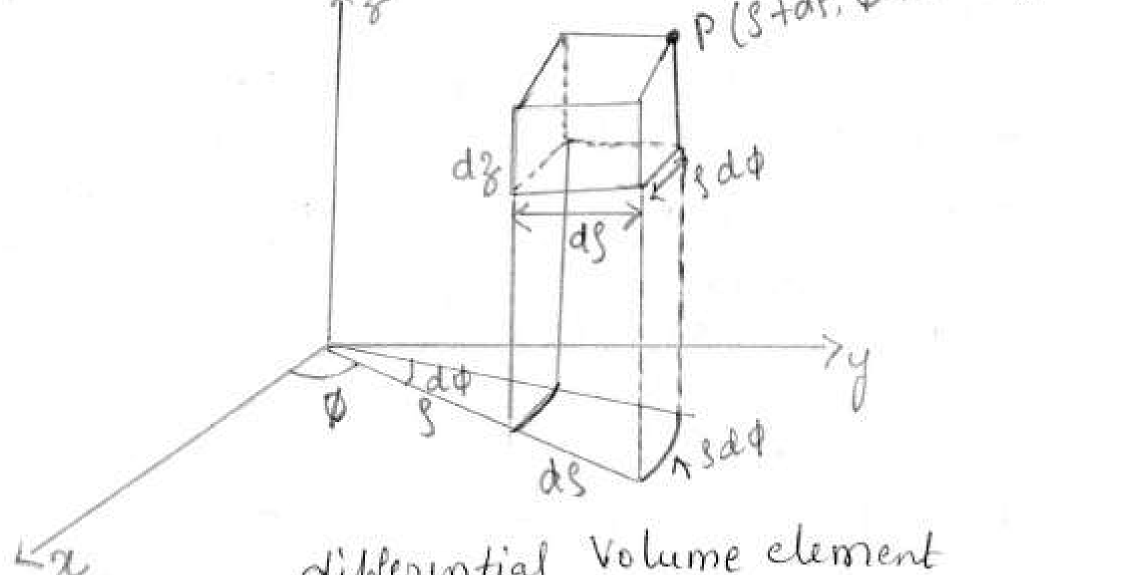
$\vec{ds_z} = \rho\, d\phi\, d\rho\, \hat{a}_z \rightarrow z$ Constant

$\therefore$ the differential lengthis

$$\boxed{\vec{dl} = d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z}$$

the magnitude of the differential length vector is given by.

$$\boxed{|\vec{dl}| = \sqrt{(d\rho)^2 + (\rho\, d\phi)^2 + (dz)^2}}$$



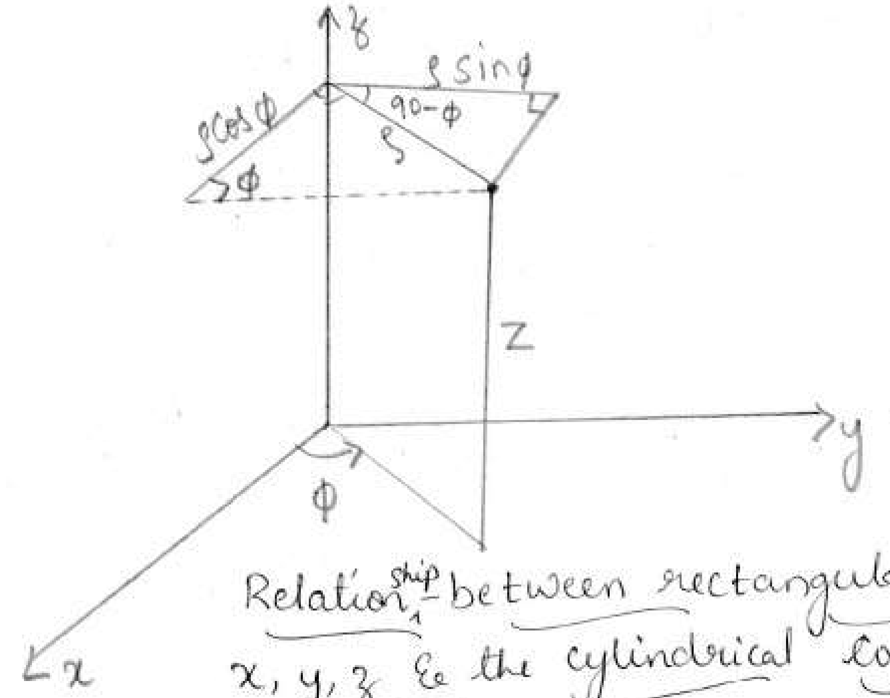
differential Volume element

The variables of the rectangular & cylindrical Co-ordinate system are related to each other by the relations

$$x = \rho \cos\phi$$
$$y = \rho \sin\phi$$
$$z = z$$

if we express cylindrical variables in-terms of x, y and z as

$$\rho = \sqrt{x^2 + y^2}\ ,\ \phi = \tan^{-1} y/x\ ,\ z = z$$



Relationship between rectangular variables x, y, z & the cylindrical Co-ordinate variables ρ, ϕ, z.

Transformation from cylindrical Co-ordinate to Cartesian.

Let vector $\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$ in Cartesian Co-ordinate

& another vector $\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$ in cylindrical Co-ordinate

to get component $A_x = \vec{A} \cdot \hat{a}_x$

$$A_x = (A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z) \cdot \hat{a}_x$$

$$= A_\rho \cos\phi + A_\phi \cos(90^\circ + \phi) + A_z (0)$$

$$= A_\rho \cos\phi - A_\phi \sin\phi$$

$$= A_\rho \frac{x}{\rho} - A_\phi \frac{y}{\rho}$$

$$\boxed{A_x = A_\rho \frac{x}{\sqrt{x^2+y^2}} - A_\phi \frac{y}{\sqrt{x^2+y^2}}}$$

111^{ly}

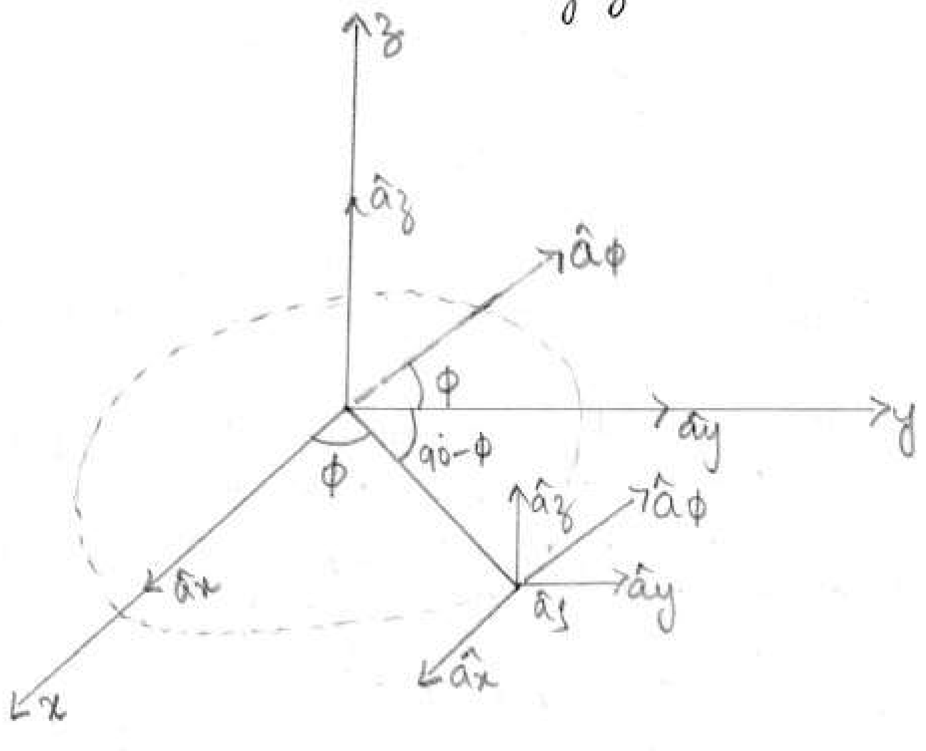
$$A_y = \vec{A} \cdot \hat{a}_y$$

$$= A_\rho \sin\phi + A_\phi \cos\phi \qquad \& \; A_z = A_z$$

$$A_y = A_\rho \frac{y}{\sqrt{x^2+y^2}} + A_\phi \frac{x}{\sqrt{x^2+y^2}}$$

$\therefore$ in Cartesian Co-ordinate

$$\vec{A} = \left[A_\rho \frac{x}{\sqrt{x^2+y^2}} - A_\phi \frac{y}{\sqrt{x^2+y^2}}\right]\hat{a}_x + \left[A_\rho \frac{y}{\sqrt{x^2+y^2}} + A_\phi \frac{x}{\sqrt{x^2+y^2}}\right]\hat{a}_y + A_z \hat{a}_z$$



	$\hat{a}_\rho$	$\hat{a}_\phi$	$\hat{a}_z$
$\hat{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\hat{a}_y$	$\sin\phi$	$\cos\phi$	0
$\hat{a}_z$	0	0	1

$\hat{a}_x \cdot \hat{a}_\rho = 1 \cdot 1 \cdot \cos\phi = \cos\phi$

$\hat{a}_y \cdot \hat{a}_\rho = \cos(90-\phi) = \sin\phi$

$\hat{a}_\rho \cdot \hat{a}_z = \cos 90^\circ = 0$

$\hat{a}_x \cdot \hat{a}_\phi = \cos(90+\phi) = -\sin\phi$

$\hat{a}_y \cdot \hat{a}_\phi = \cos\phi$

$\hat{a}_z \cdot \hat{a}_\phi = \cos 90^\circ = 0$

$\hat{a}_x \cdot \hat{a}_z = \cos 90^\circ = 0$

$\hat{a}_y \cdot \hat{a}_z = \cos 90^\circ = 0$

$\hat{a}_z \cdot \hat{a}_z = 1$

Transformation from cartisian to cylindrical

$A_\rho = \vec{A} \cdot \hat{a}_\rho$

$= (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) \cdot \hat{a}_\rho$

$$\boxed{A_\rho = A_x \cos\phi + A_y \sin\phi}$$

$A_\phi = \vec{A} \cdot \hat{a}_\phi$

$$\boxed{A_\phi = -A_x \sin\phi + A_y \cos\phi}$$

$$\boxed{A_z = A_z}$$

1) transform a vector $\vec{B} = y\hat{a}_x - x\hat{a}_y + z\hat{a}_z$ into cylindrical co-ordinates.

Soln:- $B_\rho = \vec{B} \cdot \hat{a}_\rho = (y)(\hat{a}_x \cdot \hat{a}_y) - x(\hat{a}_y \cdot \hat{a}_\rho)$

$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi$

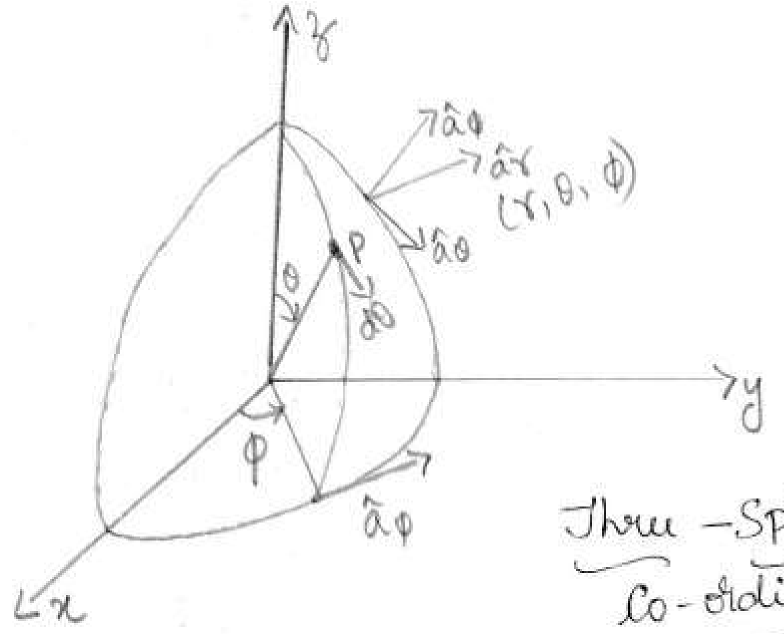
$B_\phi = \vec{B} \cdot \hat{a}_\phi = y(\hat{a}_x \cdot \hat{a}_\phi) - x(\hat{a}_z \cdot \hat{a}_\phi)$

$= -y\sin\phi - x\cos\phi = -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$

$$\boxed{\vec{B} = -\rho\hat{a}_\phi + z\hat{a}_z}$$

Spherical Co-ordinate System

The spherical co-ordinate system is built on a three rectangular axes.



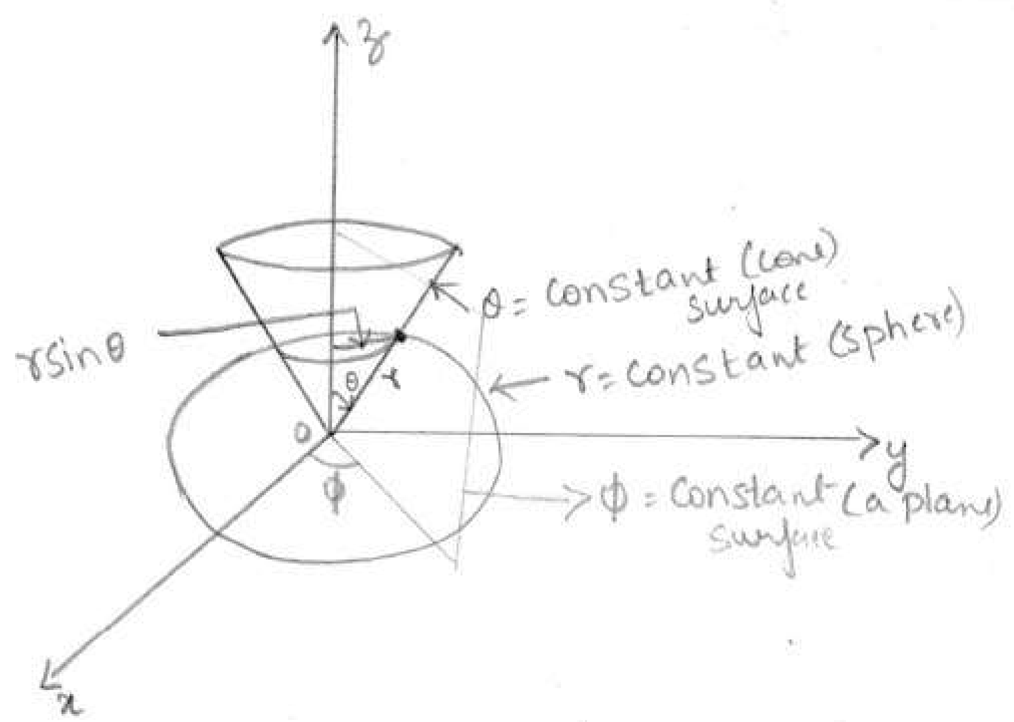
Three-Spherical Co-ordinates

From the above diagram it can be seen that the distance from origin to any point P is 'r'. The surface = constant is a sphere.

The second co-ordinate is an angle θ between the z-axis and line drawn from the origin to the point 'P'.

The surface θ = constant is a cone. The two surfaces cone & sphere are everywhere perpendicular along their intersection i.e, circle of radius '$r\sin\theta$'.

The third co-ordinate ϕ is also an angle & is exactly the same as the angle ϕ of cylindrical co-ordinates. It is the angle between the x-axis & projection in the z=0 plane of the line drawn from the origin to the point. The surface ϕ = constant is a plane passing through the $\theta = 0$ line (or z-axis).



To Construct a differential volume

Consider a point $P(r, \theta, \phi)$ in a spherical co-ordinate system. Let each co-ordinate is increased by the differential amount. The differential increments in r, θ, ϕ are $dr, d\theta, d\phi$.

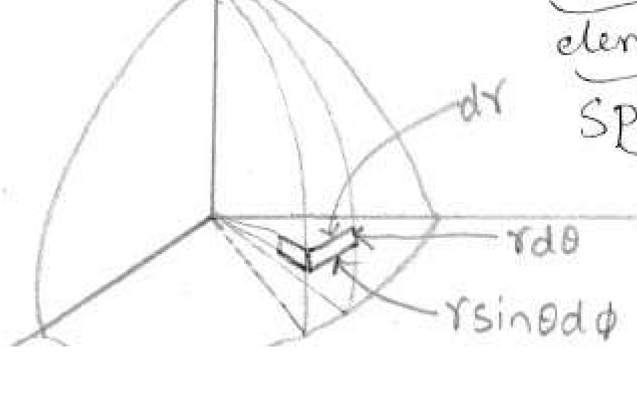
dr: differential length in 'r' direction.

$r d\theta$: differential length in 'θ' direction.

$r \sin\theta \, d\phi$: differential length in 'ϕ' direction.

Hence, the differential vector length is,

$$\boxed{\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi}$$



differential volume element in the Spherical Co-ordinate system.

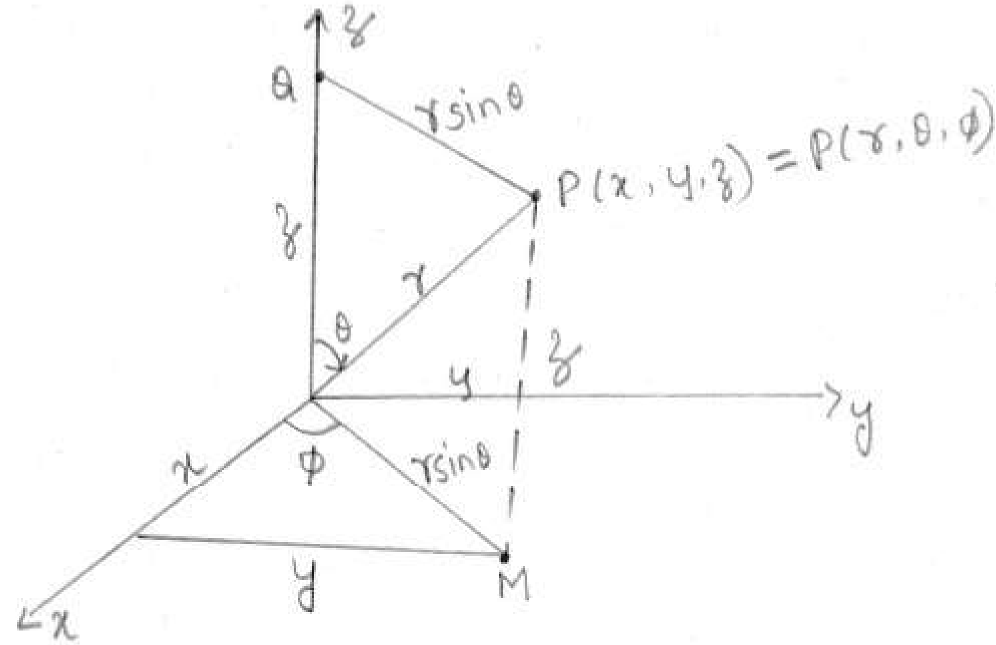
Ec differential volume is

$$\boxed{dv = r^2 \sin\theta \, dr \, d\theta \, d\phi}$$

$$\vec{dS_r} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$
$$\vec{dS_\theta} = r \sin\theta \, dr \, d\phi \, \hat{a}_\theta$$
$$\vec{dS_\phi} = r \, dr \, d\theta \, \hat{a}_\phi$$

Relationship between Cartisian Ec spherical system

Consider a point 'P' whose Cartisian Co-ordinates are x, y, z while the spherical Co-ordinates are r, θ, ϕ.



So we can write.

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$
$$\therefore \quad r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right]$$

$$\phi = \tan^{-1}(y/x)$$

$$\hat{a}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2+y^2+z^2}}$$

$$= \sin\theta\cos\phi\,\hat{a}_x + \sin\theta\sin\phi\,\hat{a}_y + \cos\theta\,\hat{a}_z$$

$$\hat{a}_\phi = \frac{\hat{a}_z \times \hat{a}_r}{\sin\theta}$$

$$= -\sin\phi\,\hat{a}_x + \cos\phi\,\hat{a}_y$$

$$\hat{a}_\theta = \hat{a}_\phi \times \hat{a}_r$$

$$= \cos\theta\cos\phi\,\hat{a}_x + \cos\theta\sin\phi\,\hat{a}_y - \sin\theta\,\hat{a}_z$$

Transformation from Cartesian vector to Spherical and vice Versa.

	$\hat{a}_r$	$\hat{a}_\theta$	$\hat{a}_\phi$
$\hat{a}_x$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{a}_y$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{a}_z$	$\cos\theta$	$-\sin\theta$	0

Cartesian to Spherical

$$A_r = \vec{A}\cdot\hat{a}_r = (A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)\cdot\hat{a}_r$$

$$\boxed{A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta}$$

$$A_\theta = \vec{A}\cdot\hat{a}_\theta = (A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)\cdot\hat{a}_\theta$$

$$\boxed{A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta}$$

$$A_\phi = \vec{A}\cdot\hat{a}_\phi = (A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)\cdot\hat{a}_\phi$$

Spherical to Cartesian

$$A_x = \vec{A} \cdot \hat{a}_x = (A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \cdot \hat{a}_\phi) \cdot \hat{a}_x$$

$$\boxed{A_x = A_r \sin\theta \cos\phi + A_\theta \cos\theta \cos\phi - A_\phi \sin\phi}$$

$$A_y = \vec{A} \cdot \hat{a}_y = (A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi) \cdot \hat{a}_y$$

$$\boxed{A_y = A_r \sin\theta \sin\phi + A_\theta \cos\theta \sin\phi + A_\phi \cos\phi}$$

$$A_z = \vec{A} \cdot \hat{a}_z = (A_r \hat{a}_r + A_\theta \hat{a}_\theta + A_\phi \hat{a}_\phi) \cdot \hat{a}_z$$

$$\boxed{A_z = A_r \cos\theta - A_\theta \sin\theta}$$

1) given the vector field $G = \frac{xz}{y} \cdot \hat{a}_x$ transform into spherical components and variables.

Soln:-

$$G_r = G \cdot \hat{a}_r = \frac{xz}{y} \hat{a}_x \cdot \hat{a}_r = \frac{xz}{y} \sin\theta \cos\phi$$

$$= r \sin\theta \cos\theta \frac{\cos^2\phi}{\sin\phi}$$

$$G_\theta = G \cdot \hat{a}_\theta = \frac{xz}{y} \hat{a}_x \cdot \hat{a}_\theta = \frac{xz}{y} \cos\theta \cos\phi$$

$$= r \cos^2\theta \frac{\cos^2\phi}{\sin\phi}$$

$$G_\phi = G \cdot \hat{a}_\phi = \frac{xz}{y} \hat{a}_x \cdot \hat{a}_\phi = \frac{xz}{y} (-\sin\phi)$$

$$= -r \cos\theta \cos\phi$$

$$\therefore \vec{G} = r \cos\theta \cos\phi \left(\sin\theta \cot\phi \, \hat{a}_r + \cos\theta \cot\phi \, \hat{a}_\theta - \hat{a}_\phi\right)$$

a. Coulomb's Law & Electric field Intensity & Flux Density

Topics Covered

Experimental law of Coulomb

Electric field intensity

Field due to Continuous Volume charge distribution

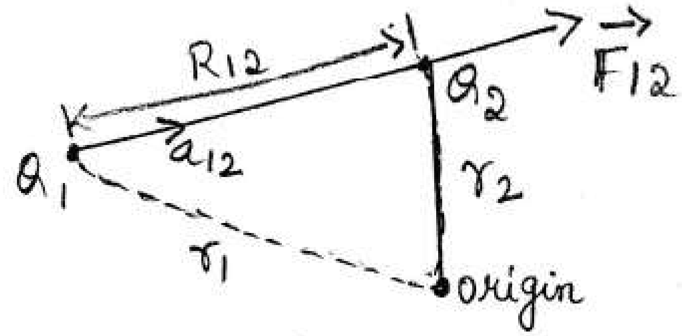
Field of a line charge.

Numericals.

Electric flux density

The law of Coulomb

It states that force between two point charges is directly proportional to product of charges & inversely proportional to square of the distance between them.



$$F = K \cdot \frac{Q_1 Q_2}{R^2} \ (N)$$

where Q_1 & Q_2 are positive / Negative quantities of charge (C),

$R \rightarrow$ is the distance between them (mts).

Constant of proportionality,

$$K = \frac{1}{4\pi\varepsilon_0}$$

$\varepsilon_0 \rightarrow$ Permittivity of free space.

$$\varepsilon_0 = 8.854 \times 10^{-12} = \frac{1}{36\pi} \times 10^{-9} \ F/m$$

Force is a vector quantity

$$\boxed{\vec{F_{12}} = \frac{9\times10^9}{\varepsilon_r} \cdot \frac{Q_1 Q_2}{|\vec{R_{12}}|^2} \hat{a_{12}}} \ N.$$

$$\hat{a_{12}} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}$$

similarly vector force acting on Q_1 due to Q_2

$$\vec{F_{21}} = \frac{9\times10^9}{\varepsilon_r} \frac{Q_1 Q_2}{|\vec{R_{21}}|^2} \hat{a_{21}} \ N.$$

$$\hat{a_{21}} = \frac{\vec{R_{21}}}{|\vec{R_{21}}|}$$

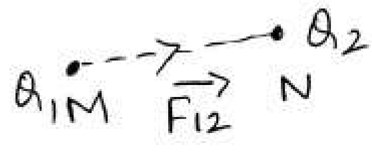
$$\Rightarrow \boxed{\vec{F_{21}} = -\vec{F_{12}}} \ \& \ \boxed{|\vec{F_{12}}| = |\vec{F_{21}}|}$$

1) Find the force exerted on Q_2 by Q_1. If a charge of $Q_1 = 3\times10^{-4}$ c at $M(1,2,3)$ & a charge of $Q_2 = -10^{-4}$ c at $N(2,0,5)$ in a Vaccum.

Soln: $\vec{R_{12}} = \vec{r_2} - \vec{r_1} = (2-1)\hat{a_x} + (0-2)\hat{a_y} + (5-3)\hat{a_z}$

$= \hat{a_x} - 2\hat{a_y} + 2\hat{a_z}$

$|\vec{R_{12}}| = \sqrt{1^2+2^2+2^2} = 3$

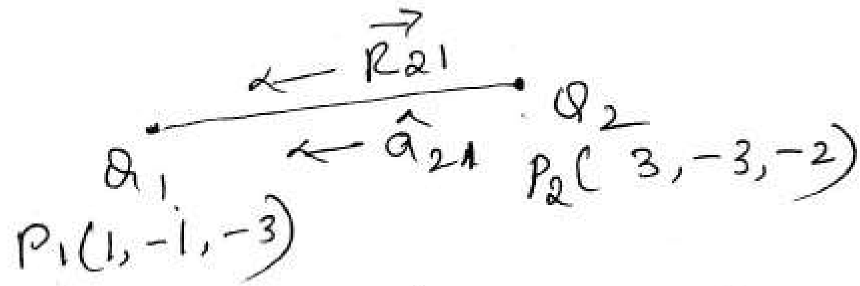


$$\vec{F_{12}} = \frac{9\times10^{9}\times 3\times10^{-4}\times(-10^{-4})}{(3)^2}\times\frac{1}{3}(\hat{a_x}-2\hat{a_y}+2\hat{a_z})$$

$$= -10\hat{a_x} \underline{+20}\,\hat{a_y} - 20\hat{a_z}\ N$$

---

2) A point charge $Q_1 = 300\mu C$ is located at $(1,-1,-3)m$ experiences a force $\vec{F} = 8\hat{a_x} - 8\hat{a_y} + 4\hat{a_z}$ N due to point charge Q_2 at $(3,-3,-2)m$. Determine Q_2



$$\vec{R_{21}} = (1-3)\hat{a_x} + (-1+3)\hat{a_y} + (-3+2)\hat{a_z}$$

$$= -2\hat{a_x} + 2\hat{a_y} - \hat{a_z} \qquad (\vec{r_1} - \vec{r_2})$$

$$|\vec{R_{21}}| = \sqrt{2^2+2^2+1^2} = \underline{3}$$

$$\hat{a_{21}} = \frac{1}{3}(-2\hat{a_x} + 2\hat{a_y} - \hat{a_z})$$

$$\vec{F_{21}} = 9\times10^{9}\times\frac{Q_1 Q_2}{|\vec{R_{21}}|^2}\hat{a_{21}}$$

$$8\hat{a_x} - 8\hat{a_y} + 4\hat{a_z} = \frac{9\times10^{9}\times 300\times10^{-6}\times Q_2}{3^2}$$

$$\times\frac{1}{3}(-2\hat{a_x}+2\hat{a_y}-\hat{a_z})$$

By equating co-efficients of $\hat{a}_x$

$$Q_2 = \frac{8}{-2\times10^5} = -4\times10^{-5}$$

$$= -40\times10^{-6}C = -\underline{\underline{40\mu c}}$$

3) A charge $Q_A = -20\mu c$ is located at A $(-6, 4, 7)$ m & a charge $Q_B = 50\mu c$ at B $(5, 8, -2)$ m in free space

find (a) $\vec{R}_{AB}$ (b) $|\vec{R}_{AB}|$ (c) Vector force exerted on Q_A by Q_B.

Soln:

$$\vec{R}_{AB} = (5+6)\hat{a}_x + (8-4)\hat{a}_y + (-2-7)\hat{a}_z$$

$$= 11\hat{a}_x + 4\hat{a}_y - 9\hat{a}_z$$

$$|\vec{R}_{AB}| = \sqrt{11^2+4^2+9^2} = \sqrt{218} = 14.76\,m$$

$$\vec{a}_{AB} = \frac{11\hat{a}_x + 4\hat{a}_y - 9\hat{a}_z}{\sqrt{218}}$$

$$\vec{F}_{BA} = -\frac{20\times10^{-6}\times50\times10^{-6}}{|\sqrt{218}|^2} \times 9\times10^9 \times \frac{-(11\hat{a}_x + 4\hat{a}_y - 9\hat{a}_z)}{\sqrt{218}}$$

$$= 30.76\hat{a}_x + 11.184\hat{a}_y - 25.1\hat{a}_z \text{ mN}$$

Electric Field Intensity

Consider a charge Q_1. If any other similar charge Q_2 is brought near it, Q_2 experiences a force. Infact if Q_1 is moved around Q_1, still Q_2 experiences a force.

Thus there exists a region around a charge in which it exerts a force on any other charge.

This region where a particular charge exerts a force on any other charge located in that region is called electric field

The force experienced by the charge Q_2 due to Q_1 is given by Coulomb's law as,

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |R_{12}^2|} \hat{a}_{12}$$

thus force per unit charge can be written as,

$$\frac{\vec{F_2}}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0 |\vec{R}_{12}^2|} \hat{a}_{12}$$

∴ This force exerted per unit charge is called electric field Intensity or

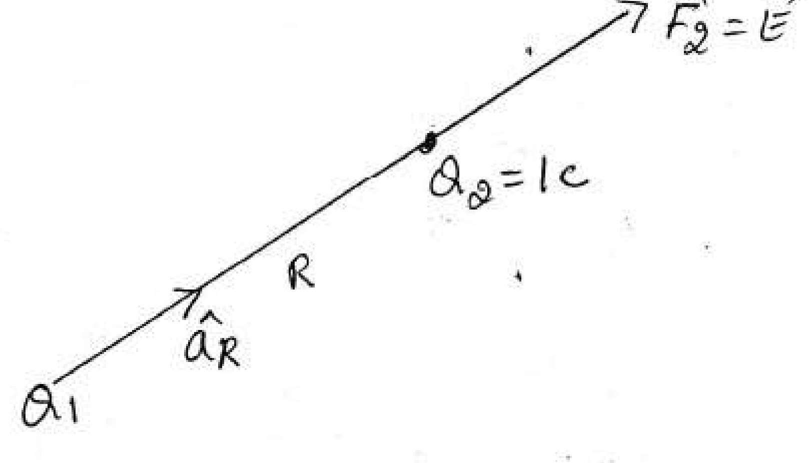
[illegible] vector Quantity

$$\therefore \vec{E} = \frac{Q_1}{4\pi\varepsilon_0 |R_{1p}^2|} \hat{a}_{1p} \quad N/C \text{ or } V/m$$

P= position of any other charge around Q_1.

the above equation is the electric field intensity due to a single point charge Q_1 in a free space or vaccum.

Consider a charge Q_1 as shown below



the unit positive charge $Q_2 = 1c$ is placed at a distance R from Q_1. Then the force acting on Q_2 due to Q_1 is along the unit vector $\hat{a}_R$. As the charge Q_2 is unit charge, the force exerted on Q_2 is nothing but electric field intensity $\vec{E}$ of Q_1 at the point where unit charge is placed.

$$\therefore \quad \vec{E} = \frac{Q_1}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

R = magnitude of the Vector R.

Representation in Rectangular Co-ordinate System.

Case 1:

Consider a charge Q at the origin.

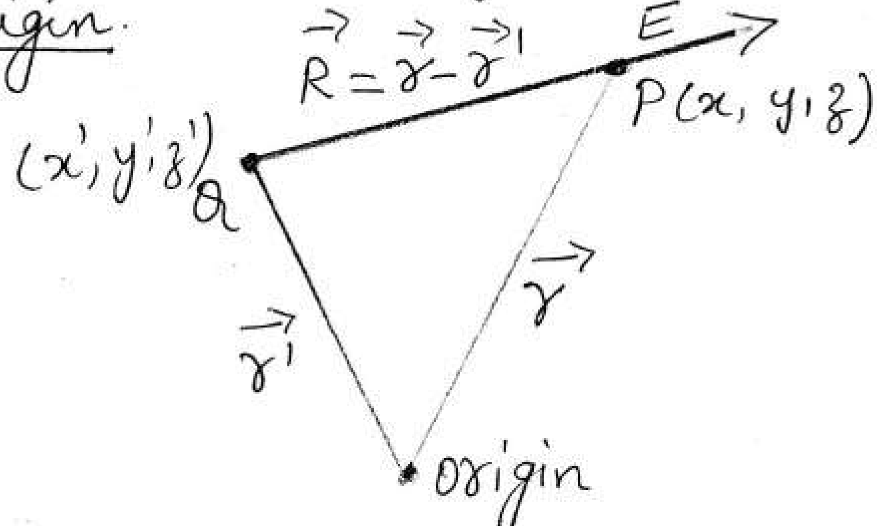
if $\vec{R} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

$$\hat{a}_R = \frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2+y^2+z^2}}$$

$$\therefore \quad \vec{E} = \frac{Q}{4\pi\varepsilon_0(x^2+y^2+z^2)}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{a}_x + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{a}_y + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{a}_z\right)$$

Case 2:

Consider a charge which is not at the origin.



for a charge Q located at the source Point

$$\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$\vec{r}\,' = x'\hat{a}_x + y'\hat{a}_y + z'\hat{a}_z$$

$$\therefore \vec{R} = \vec{r} - \vec{r}\,' = (x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z$$

$\therefore$ Electric field at Point 'P' is

$$\vec{E}_P = \frac{Q}{4\pi\varepsilon_0 |\vec{r}-\vec{r}\,'|^2} \times \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left\{\frac{(x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}}\right\}$$

Case 3:
Electric field due to two point charges.

Since, the Coulomb forces are linear, the electric field intensity due to two point charges Q_1 at r_1 & Q_2 at r_2 is the sum of the forces on Q_t caused by Q_1 and Q_2 acting alone.

$$\therefore \vec{E} = \frac{Q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^2}\hat{a}_1 + \frac{Q_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^2}\vec{a}_2$$

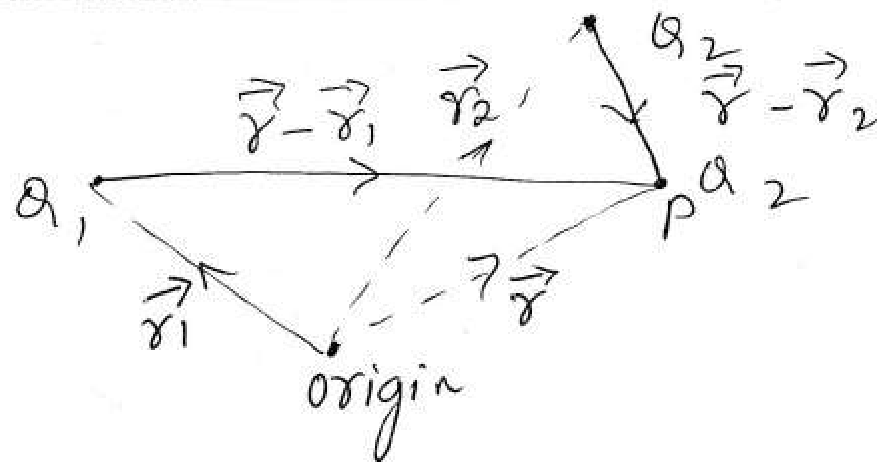
$\hat{a}_1, \hat{a}_2 \Rightarrow$ are unit vectors in the direction of $(\vec{r}-\vec{r}_1)$ & $(\vec{r}-\vec{r}_2)$ resp.

If we add more charges at other positions, the field due to n point charges is

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^2}\hat{a}_1 + \frac{Q_2}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^2}\hat{a}_2 + \cdots + \frac{Q_n}{4\pi\varepsilon_0|\vec{r}-\vec{r}_n|^2}\hat{a}_n$$

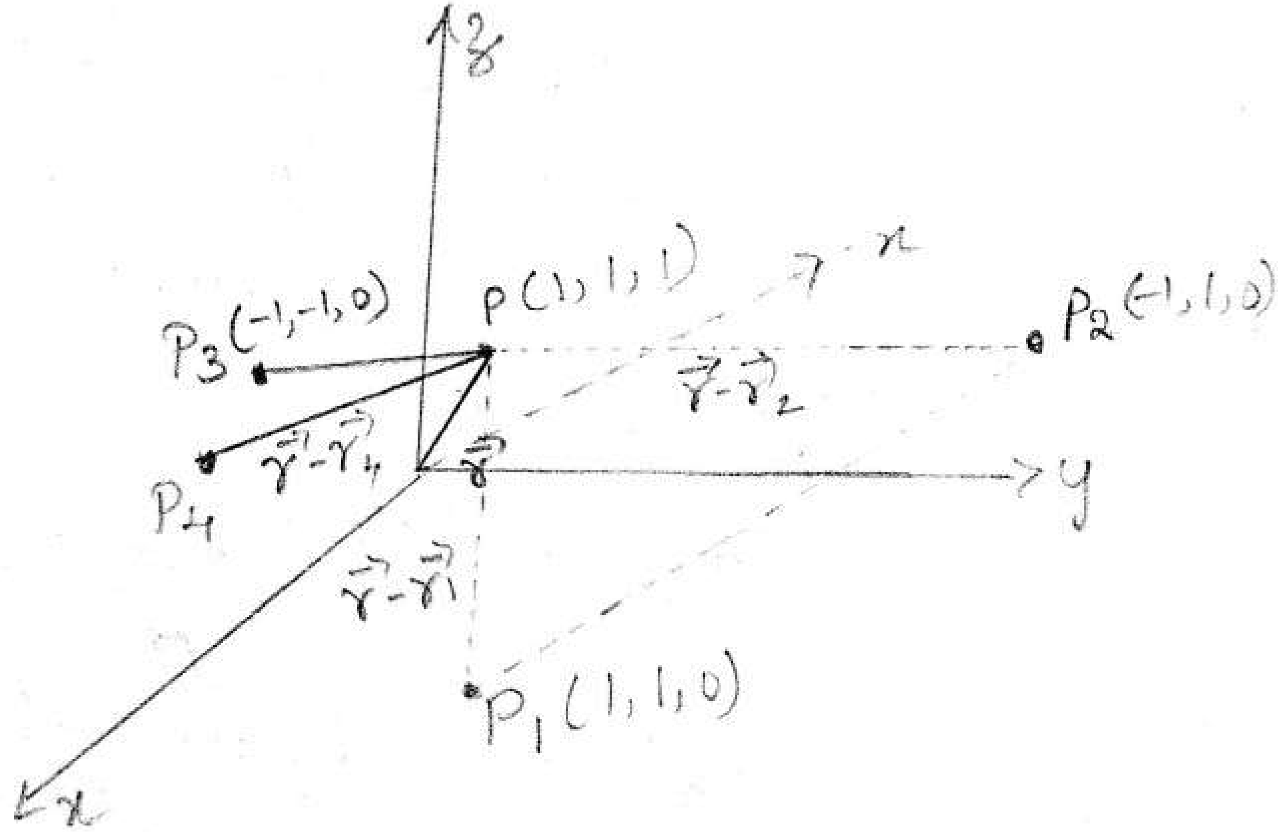
OR

$$\boxed{\vec{E} = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0|\vec{r}-\vec{r}_m|^2}\hat{a}_m}$$



Problems

1) Find $\vec{E}$ at point (1, 1, 1) caused by four identical 3 nc (nano coulomb) charges located at P_1 (1, 1, 0), P_2 (−1, 1, 0), P_3 (−1, −1, 0) & P_4 (+1, −1, 0).



$$\vec{r}_{1P} = \vec{r} - \vec{r}_1 = (1-1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z$$
$$= \hat{a}_z$$
$$\vec{r}_{2P} = \vec{r} - \vec{r}_2 = (1+1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z$$
$$= 2\hat{a}_x + \hat{a}_z$$
$$\vec{r}_{3P} = \vec{r} - \vec{r}_3 = (1+1)\hat{a}_x + (1+1)\hat{a}_y + (1-0)\hat{a}_z$$
$$= 2\hat{a}_x + 2\hat{a}_y + \hat{a}_z$$
$$\vec{r}_{4P} = \vec{r} - \vec{r}_4 = (1-1)\hat{a}_x + (1+1)\hat{a}_y + (1-0)\hat{a}_z$$
$$= 2\hat{a}_y + \hat{a}_z$$
$$\therefore \vec{E}_P = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\hat{a}_{1P}}{r_{1P}^2} + \frac{\hat{a}_{2P}}{r_{2P}^2} + \frac{\hat{a}_{3P}}{r_{3P}^2} + \frac{\hat{a}_{4P}}{r_{4P}^2}\right]$$
$$= \frac{3\times 10^{9}}{4\pi \times \frac{1}{36\pi}\times 10^{-9}}\left[\frac{\hat{a}_z}{1.1^2} + \frac{2\hat{a}_x + \hat{a}_z}{\sqrt{5}}\cdot\left(\frac{1}{\sqrt{5}}\right)^2\right.$$
$$\left. + \frac{2\hat{a}_x + 2\hat{a}_y + \hat{a}_z}{3}\cdot\frac{1}{3^2} + \frac{2\hat{a}_y + \hat{a}_z}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}\right]$$
$$\vec{E}_P = 6.82\hat{a}_x + 6.82\hat{a}_y + 32.8\hat{a}_z \ \text{V/m}$$

2) A charge of $-0.3\,\mu C$ is located at $A(25, -30, 15)$ in (cm) & a second charge of $0.5\,\mu C$ is at $B(-10, 8, +12)$ cm find $\vec{E}$ at (a) origin (b) $P(15, 20, 50)$ cm

(a) at origin [$P(0,0,0)$]

$$\vec{r}_{AP} = (-25\hat{a}_x + 30\hat{a}_y - 15\hat{a}_z) \times 10^{-2}\, m$$

$$\vec{r}_{BP} = (10\hat{a}_x - 8\hat{a}_y - 12\hat{a}_z \times 10^{-2}\, m$$

$$\hat{a}_{AP} = \frac{-25\hat{a}_x + 30\hat{a}_y - 15\hat{a}_z}{\sqrt{(25)^2 + (30)^2 + (15)^2}} \times 10^{-2}$$

$$\hat{a}_{BP} = \frac{10\hat{a}_x - 8\hat{a}_y - 12\hat{a}_z}{\sqrt{10^2 + 8^2 + 12^2} \times 10^{-2}}$$

$$\vec{E}_P = -0.3 \times 10^{-6} \times 9 \times 10^{9} \left[\frac{-25\hat{a}_x + 30\hat{a}_y - 15\hat{a}_z}{} \right]$$

$$+ 0.5 \times 10^{-6} \times 9 \times 10^{9} \left[10\hat{a}_x - 8\hat{a}_y - 15\hat{a}_z \right]$$

$$= 92.3\hat{a}_x \underline{-77}.6\hat{a}_y - 94.2\hat{a}_z \quad KV/m$$

(b) at $P(15, 20, 50)$ cm

$$\vec{r}_{AP} = (-10\hat{a}_x + 50\hat{a}_y + 35\hat{a}_z) \times 10^{-2}\, m$$

$$\vec{r}_{BP} = (25\hat{a}_x + 12\hat{a}_y + 38\hat{a}_z) \times 10^{-2}\, m$$

$$\hat{a}_{AP} = \frac{-10\hat{a}_x + 50\hat{a}_y + 35\hat{a}_z}{\sqrt{(10)^2 + (50)^2 + (35)^2}} \times \frac{10^{-2}}{10^{-2}}$$

$$\hat{a}_{BP} = \frac{25\hat{a}_x + 12\hat{a}_y + 38\hat{a}_z}{\sqrt{(25)^2 + (12)^2 + (38)^2}} \times \frac{10^{-2}}{10^{-2}}$$

$$\vec{E}_P = -0.3 \times 10^{-6} \times 9 \times 10^9 \left[\frac{-10\hat{a}_x + 50\hat{a}_y + 35\hat{a}_z}{(3825)^{3/2}}\right]$$

$$+ 0.5 \times 10^{-6} \times 9 \times 10^9 \left[\frac{25\hat{a}_x + 12\hat{a}_y + 38\hat{a}_z}{(2213)^{3/2}}\right]$$

$$= 11.9\hat{a}_x \underline{\underline{-0.519}}\,\hat{a}_y + 124\hat{a}_z \text{ KV/m}$$

Field due to Continuous Volume charge distribution

If a region of space filled with tremendous no. of charges separated by minute distances and denote this Volume charge density by ρ_v C/m^3 (coulombs/cubic meter).

The small amount of charge ΔQ in small Δv Volume is,

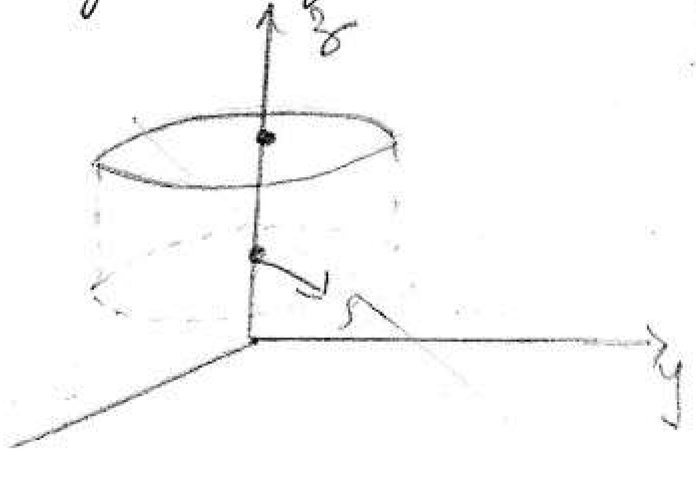
$$\Delta Q = \rho_v \Delta v$$

& ρ_v mathematically is defined using limiting process as

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

The total charge within some finite Volume is obtained by integrating the Volume.

$$\boxed{Q = \int_{vol} \rho_v \, dv}$$



The incremental Electric field intensity at $\vec{r}$ produced by an incremental charge ΔQ at $\vec{r}'$ is

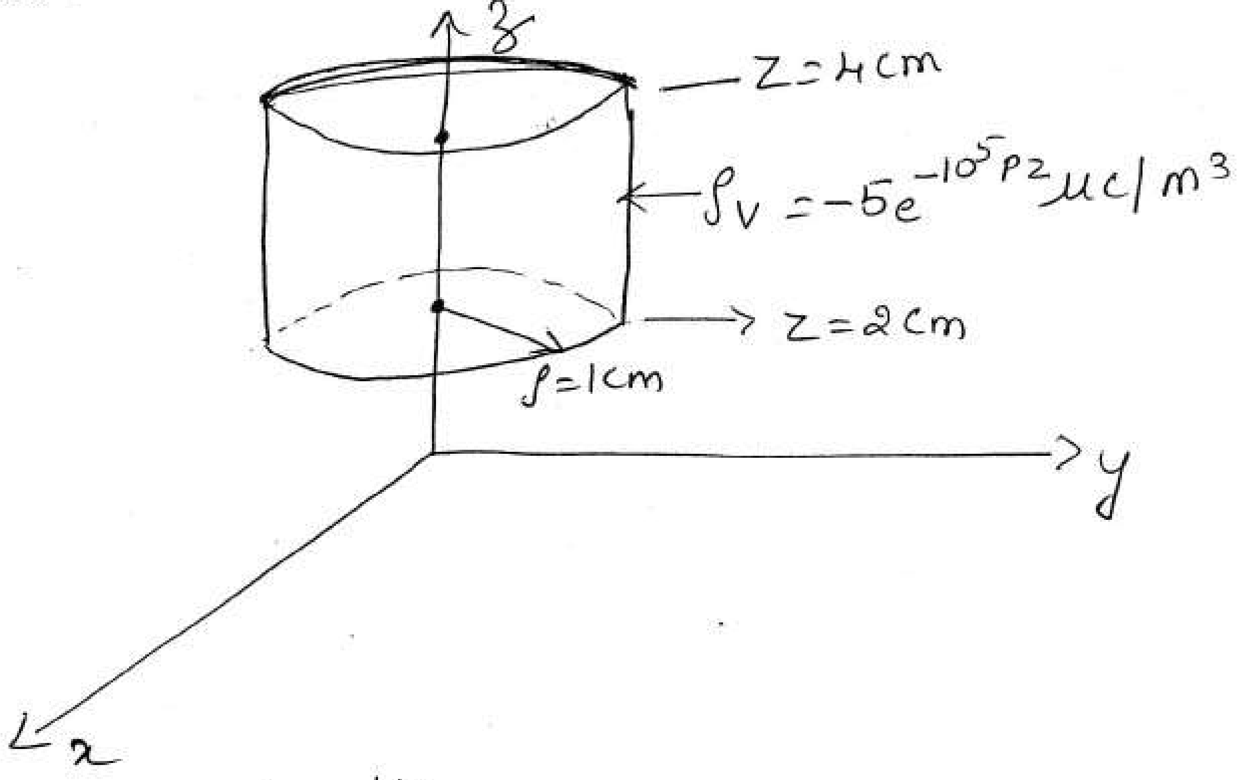
$$\Delta \vec{E} = \frac{\Delta Q}{4\pi \varepsilon_0 |\vec{r}-\vec{r}'|^2} \cdot \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$$

$$= \frac{\rho_v \Delta v}{4\pi \varepsilon_0 |\vec{r}-\vec{r}'|^2} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$$

By summing the contribution of all the volume charge in a given region, & let $\Delta v \to 0$, as no of elements become infinite, the summation becomes integral.

$$\boxed{\vec{E} = \int_{vol} \frac{\rho_v \, dv}{4\pi \varepsilon_0 |\vec{r}-\vec{r}'|^2} \cdot \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^2}}$$

1) Find the total charge contained in a 2-cm length of the element beam shown below.



charge density

$$\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z}\ C/m^2$$

$$Q = \int_{vol} \rho_v \, dv$$

The Volume differential in cylindrical co-ordinate

$$Q = \int_{z=0.02}^{0.04} \int_{\phi=0}^{2\pi} \int_{\rho=0}^{0.01} -5 \times 10^{-6} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

$$= -2\pi \times 5 \times 10^{-6} \int_{z=0.02}^{0.04} \int_{\rho=0}^{0.01} \rho \, d\rho \, dz \left(e^{-10^5 \rho z} \right)$$

$$= -10^{-5}\pi \int_0^{0.01} \left[\frac{1}{-10^5\rho} e^{-10^5 \rho z} \rho \, d\rho \right]_{z=0.02}^{z=0.04}$$

$$= \int_0^{0.01} -10^{-5}\pi \left(e^{-2000\rho} - e^{-4000\rho} \right) d\rho$$

$$Q = -10^{-10}\pi \left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{-4000} \right)_0^{0.01}$$

$$Q = -10^{-10}\pi \left(\frac{1}{2000} - \frac{1}{4000} \right) = \frac{-\pi}{40} = \underline{\underline{0.0785 pC}}$$

---

(5) Find the element beam current, if these elements are moving at a Constant Velocity of 10% of velocity of light, & if it travels a distance of 2cm.

Soln:
Velocity of $el^- = \frac{10}{100} \times 3\times10^8 = 3 \times 10^7$ m/sec

time taken to travel 2cm,

$$\Delta t = \frac{2\times10^{-2}}{3\times10^7} = \frac{2}{3}\times10^{-9}$$

$$\therefore I = \frac{\Delta Q}{\Delta t} = \frac{-(\pi/40)\times10^{-12}}{(2/3)\times10^{-9}} = \underline{\underline{118 \mu A}}$$

2) Calculate the total charge within each of the indicated volume (a) $0.1 \leq |x|, |y|, |z| \leq 0.2$

$$\rho_v = \frac{1}{x^3 y^3 z^3}$$

<u>soln</u>

$$Q = \int_{vol} \rho_v \, dv$$

$$= \int_x \int_y \int_z \frac{1}{x^3 y^3 z^3} dx\,dy\,dz$$

$$= \int_{x=0.1}^{0.2} \int_{y=0.1}^{0.2} \int_{z=0.1}^{0.2} \frac{1}{x^3 y^3 z^3} dx\,dy\,dz$$

$$+ \int_{x=-0.2}^{-0.1} \int_{y=-0.2}^{-0.1} \int_{z=0.2}^{-0.1} \frac{1}{x^3 y^3 z^3} dx\,dy\,dz$$

$$= \left.\frac{x^{-2}}{-2}\right]_{0.1}^{0.2} \left.\frac{y^{-2}}{-2}\right]_{0.1}^{0.2} \left.\frac{z^{-2}}{-2}\right]_{0.1}^{0.2} +$$

$$\left.\frac{x^{-2}}{-2}\right]_{-0.2}^{0.1} \left.\frac{y^{-2}}{-2}\right]_{-0.2}^{-0.1} \left.\frac{z^{-2}}{-2}\right]_{0.1}^{0.2}$$

$$= -\frac{1}{8}\left[(0.2)^{-6} - (0.1)^{-6}\right] - \frac{1}{8}\left[(-0.1)^{-6} - (-0.2)^{-6}\right]$$

(b) $0 \leq \rho \leq 0.1, \quad 0 \leq \phi \leq \pi, \quad 2 \leq z \leq 4$

$\rho_v = \rho^2 z^2 \sin 0.6\phi$

$$Q = \int_{vol} \rho_v \, dv$$

$$= \int_{\rho=0}^{0.1} \int_{\phi=0}^{\pi} \int_{z=2}^{4} \rho^2 z^2 \sin 0.6\phi \; \rho \, d\rho \, d\phi \, dz$$

$$= \left[\frac{\rho^4}{4}\right]_0^{0.1} \left[-\frac{\cos 0.6\phi}{0.6}\right]_0^{\pi} \left[\frac{z^3}{3}\right]_2^4$$

$$= \left[\frac{(0.1)^4}{4} - 0\right] \left[\frac{0.309}{0.6} + \frac{1}{0.6}\right] \left[\frac{4^3}{3} - \frac{2^3}{3}\right]$$

$$= \left[0.000025 \times 2.18\right] \times \left[18.66\right]$$

$$= \underline{\underline{1.018 \, mc}}$$

(c) $\rho_v = \dfrac{e^{-2r}}{r^2}$, universe

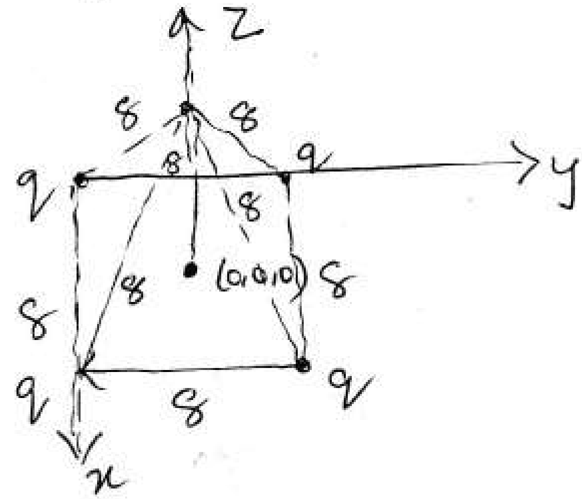
$$Q = \int_{r_0}^{\infty} \int_{\theta_0}^{\pi} \int_{\phi_0}^{2\pi} \frac{e^{-2r}}{r^2} \, r^2 \, dr \sin\theta \, d\theta \, d\phi$$

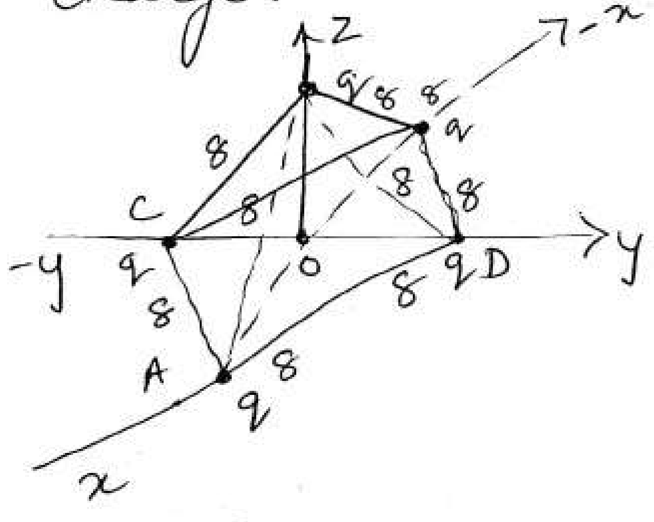
$$= 2\pi \left[-\cos\theta\right]_0^{\pi} \left[\frac{e^{-2r}}{-2}\right]_0^{\infty}$$

$$= \frac{-2\pi}{-2}\left[-1-1\right]\left[\frac{e^{-\infty}}{+1} - \frac{e^{-0}}{1}\right]$$

$$= -2\pi\left[-1\right] = \underline{\underline{6.28C}}$$

3) four 10nc +ve charges are located in Z=0 plane at the corners of a square, 8cm on a side. A fifth 10nc +ve charge is located at a point 8cm distance from other charges. Calculate the magnitude of force on this 5th charge.





$$d_{CD} = \sqrt{8^2+8^2} = 11.31$$

$$OD = OC = OA = OB = \frac{11.31}{2} = 5.66\text{cm}$$

$\therefore$ Co-ordinate $A\ (5.66, 0, 0)$

$B\ (-5.66, 0, 0)$

$C\ (0, -5.66, 0)$

$D\ (0, 5.66, 0)$ cm

distance $OP = \sqrt{8^2 - (5.66)^2} = 5.65$ cm

Co-ordinate of $P\ (0, 0, 5.65)$ cm

total force acting on charge Q placed at P,

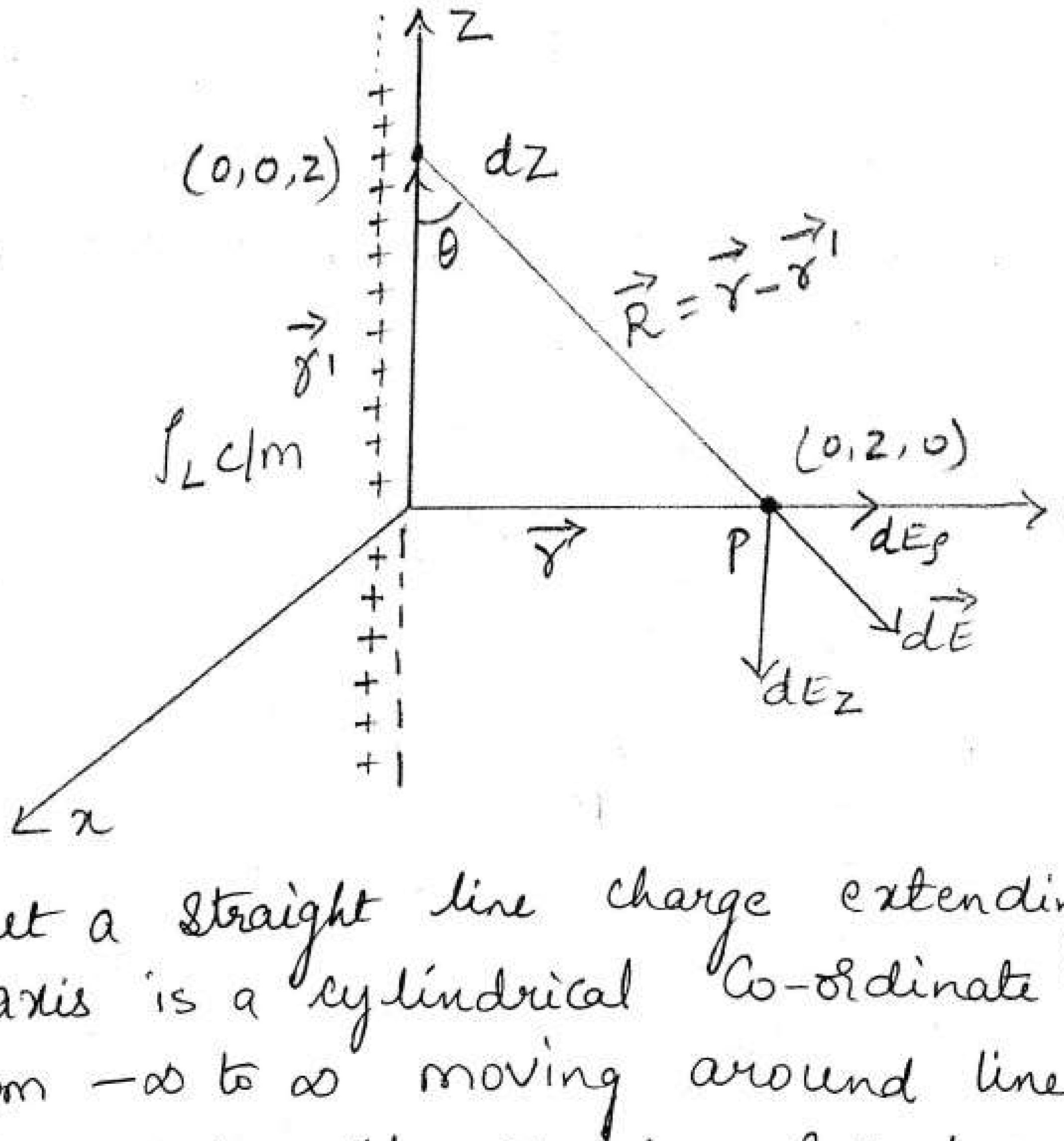
$$\vec{F}_P = \vec{F}_{AP} + \vec{F}_{CP} + \vec{F}_{BP} + \vec{F}_{DP}$$

$$= 9\times10^9 \left[\frac{Q^2}{|\vec{r}_{AP}|^2}\hat{a}_{AP} + \frac{Q^2}{|\vec{r}_{BP}|^2}\hat{a}_{BP} + \frac{Q^2}{|\vec{r}_{CP}|^2}\hat{a}_{CP} + \frac{Q^2}{|r_{DP}|^2}\hat{a}_{DP} \right]$$

$$= 9\times10^9 \times 10\times10\times10^{-9}\times10^{-9} \left[\frac{-5.66\hat{a}_x + 5.65\hat{a}_y}{8^3} + \frac{-5.66\hat{a}_y + 5.65\hat{a}_z}{8^3} + \frac{5.66\hat{a}_x + 5.65\hat{a}_z}{8^3} + \frac{5.66\hat{a}_y + 5.65\hat{a}_x}{8^3} \right]$$

Field of a Line charge

Consider a filament like distribution of volume charge density called as <u>line charge density</u>, ρ_L c/m.



Let a straight line charge extending along Z-axis is a cylindrical Co-ordinate System from $-\infty$ to ∞ moving around line charge, varying ϕ while keeping ρ and z constant. the line charge appears same, from every angle so no field component vary with ϕ.

Again if we maintain ρ & ϕ constant while moving up & down the line charge by changing z, the line charge still recedes into infinite distance in both directions called axial symmetry. No field component vary with z.

By maintaining ϕ & z constant & vary ρ, the field becomes weaker as ρ increases.

By choosing a point $P(0, y, 0)$ on y axis to determine field,

with $dQ = \rho_L dz$

$$\vec{dE} = \frac{\rho_L dz\,(\vec{r} - \vec{r}')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}'|^3}$$

$$\vec{r} = y\hat{a_y} = \rho\hat{a}_\rho$$

$$\vec{r}' = z' a_z$$

$$\vec{R} = \rho\hat{a}_\rho - z\hat{a}_z$$

$$\therefore \vec{dE} = \frac{\rho_L dz\,(\rho a_\rho - z\hat{a}_z)}{4\pi\varepsilon_0 (\rho^2 + z^2)^{3/2}}$$

Since only the $\vec{E_\rho}$ component is present,

$$\vec{dE} = \frac{\rho_L \rho dz}{(\rho^2 + z^2)^{3/2}}\hat{a}_\rho$$

$$\vec{E} = \int_{-\infty}^{\infty} \frac{\rho_L \rho \, dz}{4\pi\varepsilon_0 (\rho^2+z^2)^{3/2}} \hat{a}_\rho$$

W.K.T from figure $\tan\theta = \frac{\rho}{z}$

or $z = \rho\cot\theta$

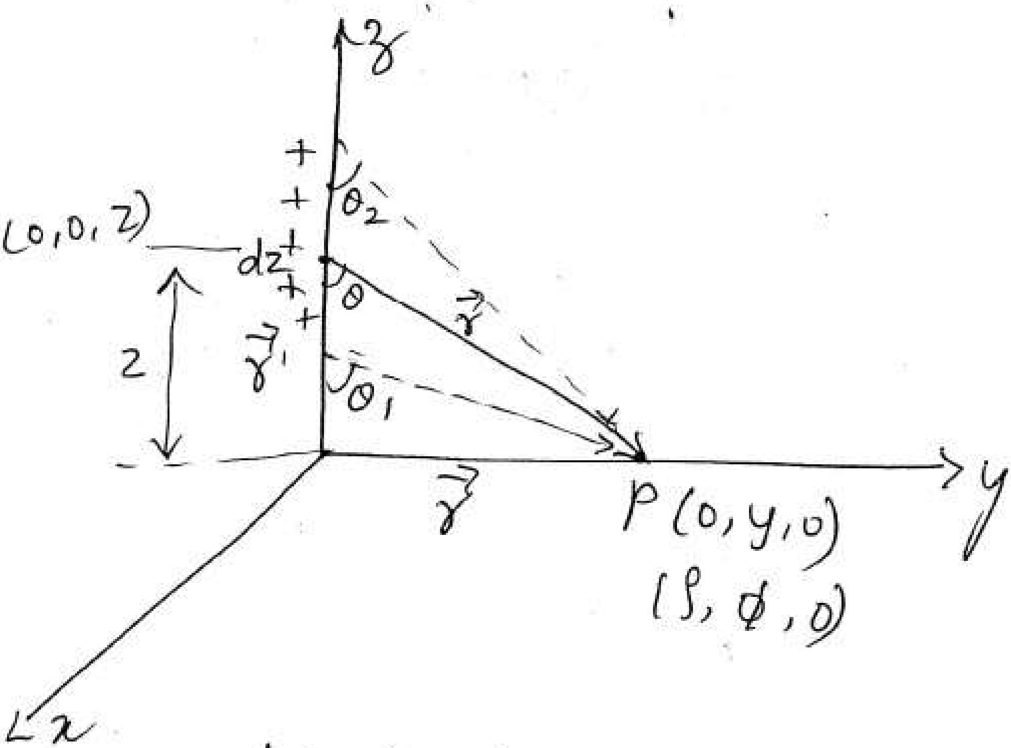
$$\therefore \vec{E} = \int_{\pi}^{0} \frac{\rho_L \ \rho(-\rho.\mathrm{cosec}^2\theta) \, d\rho}{4\pi\varepsilon_0 (\rho^2+\rho^2\cot^2\theta)^{3/2}} \, d\rho \, \hat{a}_\rho$$

$$= \int_{\pi}^{0} \frac{-\rho_L}{4\pi\varepsilon_0\rho} \sin\theta \, d\theta \ \hat{a}_\rho$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho} \hat{a}_\rho}$$

① Find the electric field intensity at any Point P due to a uniformly charged wire of length L m having charge density ρ_l C/m.

Soln



$$dQ = \rho_L \, dl$$
$$= \rho_l \, dz \ C$$

$$d\vec{E} = \frac{\rho_L \, dz}{4\pi\varepsilon_0} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{r} = y\hat{a}_y = \rho\hat{a}_\rho$$
$$\vec{r}' = z\hat{a}_z$$
$$\vec{R} = \vec{r} - \vec{r}' = \rho\hat{a}_\rho - z\hat{a}_z$$
$$d\vec{E} = \frac{\rho_L \, dz \, (\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi\varepsilon_0 (\rho^2 + z^2)^{3/2}}$$

from figure, $\tan\theta = \frac{\rho}{z}$; $\cot\theta = z/\rho$

$$d\vec{E} = \frac{\rho_l \, (-\rho \operatorname{cosec}^2\theta \, d\theta)}{4\pi\varepsilon_0 (\rho^3 \operatorname{cosec}^2\theta)} (\rho \hat{a}_\rho - \rho \hat{a}_z \cot\theta)$$

$$= \frac{-\rho_l \sin\theta \, d\theta}{4\pi\varepsilon_0 \cdot \rho} (\hat{a}_\rho - \cot\theta \, \hat{a}_z)$$

$$d\vec{E} = \frac{-\rho_l}{4\pi\varepsilon_0\rho} \left[\sin\theta \, \hat{a}_\rho - \cos\theta \, \hat{a}_z\right] d\theta$$

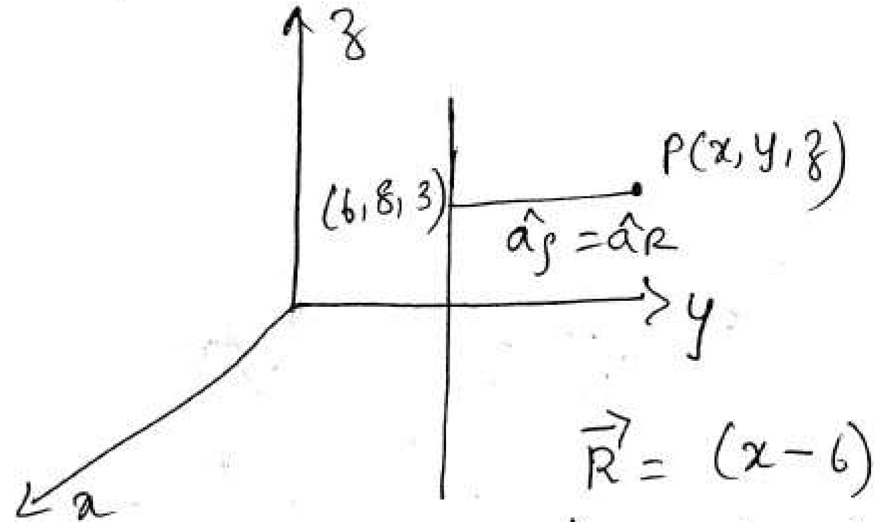
to find the total field, integrate from θ_1 to θ_2.

$$\vec{E} = \frac{-\rho_l}{4\pi\varepsilon_0\rho} \left[\int_{\theta_1}^{\theta_2} (\sin\theta \, \hat{a}_\rho - \cos\theta \, \hat{a}_z) \, d\theta\right]$$

$$= \frac{-\rho_l}{4\pi\varepsilon_0\rho} \left(\left[-\cos\theta\right]_{\theta_1}^{\theta_2} \hat{a}_\rho - \left[\sin\theta\right]_{\theta_1}^{\theta_2} \hat{a}_z \right)$$

$$= \frac{\rho_l}{4\pi\varepsilon_0\rho} \left[(\cos\theta_2 - \cos\theta_1) \hat{a}_\rho + (\sin\theta_2 - \sin\theta_1) \hat{a}_z\right] \text{ V/m}$$

① Find $\vec{E}$ at $P(x, y, z)$ if a infinite line charge is parallel to z-axis at $x=6$, $y=8$, with ρ_l c/m line charge density.



$$\vec{R} = (x-6)\hat{a}_x + (y-8)\hat{a}_y$$

$$\hat{a}_R = \frac{(x-6)\hat{a}_x + (y-8)\hat{a}_y}{\left[(x-6)^2 + (y-8)^2\right]^{1/2}}$$

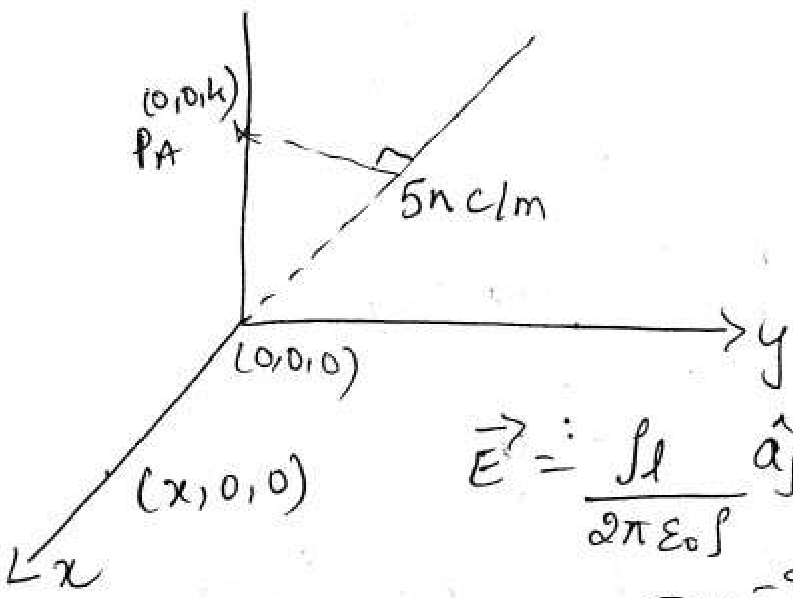
$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0\sqrt{(x-6)^2+(y-8)^2}}\hat{a}_R$$

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0}\,\frac{(x-6)\hat{a}_x+(y-8)\hat{a}_y}{\left[(x-6)^2+(y-8)^2\right]}$$

2) Infinite uniform line charge of 5nc/m lie along the (+ve & -ve) x-axis & y-axis in free space. find $\vec{E}$ at (a) $P_A(0,0,4)$ (b) $P_B(0,3,4)$

Soln:-

(a) $P_A(0,0,4)$



$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0\rho}\hat{a}_\rho$$

$$= \frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\times\frac{4\hat{a}_z}{4}$$

$$+\frac{5\times10^{-9}}{2\pi\varepsilon_0(4)}\times\frac{4\hat{a}_z}{4}$$

$= 2\times5\times10^{-9}\hat{a}_z$

(b) $P_B(0,3,4)$

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0 (\vec{R_x})}\hat{a}_R + \frac{\rho_l \, \hat{a_y}}{2\pi\varepsilon_0 |\vec{R_y}|}$$

$$\vec{R_x} = 3\hat{a_y} + 4\hat{a_z} \Rightarrow |\vec{R_x}| = 5$$

$$\vec{R_y} = 4\hat{a_z} \Rightarrow |\vec{R_y}| = 4.$$

$$\vec{E} = \frac{\cancel{5}\times 10^{-9}}{2\pi\varepsilon_0 \times 5} \frac{(3\hat{a_y} + 4\hat{a_z})}{\cancel{5}} + \frac{5\times 10^{-9}}{2\pi\varepsilon_0 \times 4} \times \frac{\cancel{4}\hat{a_z}}{\cancel{4}}$$

$$= \underline{10.8\hat{a_y} + 36.9\hat{a_z} \ \text{V/m}}$$

2) Find $\vec{E}$ at $P(1,5,2)$ in free space if a Point charge of $6\mu C$ is located at $Q(0,0,1)$. a uniform line charge of $180\,nC/m$ lies along the x-axis & a uniform Sheet of charge equal to $25\,nC/m^2$ lies in the Plane $z=-1$.

Soln $\vec{E}_1$ due point charge

$$\vec{E}_1 = \frac{Q}{4\pi\varepsilon_0 |\vec{R}_{1P}|^2} \cdot \hat{a}_{1P}$$

$$\vec{R}_{1P} = \hat{a}_x + 5\hat{a}_y + \hat{a}_z \quad , \quad |\vec{R}_{1P}| = 3\sqrt{3}$$

$$\vec{E}_1 = \frac{6\times10^{-6}}{4\pi\varepsilon_0 \times (27)^{3/2}} \times (\hat{a}_x + 5\hat{a}_y + \hat{a}_z)$$

$$= 384.7\,\hat{a}_x + 1923.7\,\hat{a}_y + 384.7\,\hat{a}_z$$

$\vec{E}_2$ due to line Charge

the radial distance b/w line charge & Point P.

$$R = \rho = \sqrt{5^2+2^2} = \sqrt{29}$$

$$\vec{E}_2 = \frac{\rho_l\,\hat{a}_\rho}{2\pi\varepsilon_0\rho} = \frac{180\times10^{-9}}{2\pi\varepsilon_0 \times\sqrt{29}} \times \frac{5\hat{a}_y + 2\hat{a}_z}{\sqrt{29}}$$

$$= 558.39\,\hat{a}_y + 223.36\,\hat{a}_z$$

$\vec{E}_3$ due to surface charge,

$$\vec{E}_3 = \frac{\rho_S \hat{a}_N}{2\varepsilon_0}$$

$$= \frac{\rho_S}{2\varepsilon_0}\hat{a}_z = \frac{25\times10^{-9}}{2\varepsilon_0}\hat{a}_z$$

$$= \underline{\underline{1412\hat{a}_z}}$$

$$\therefore \vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 384.7\hat{a}_x + 2482.09\hat{a}_y + 2020.06\hat{a}_z \text{ V/m}$$

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③ A point charge $Q_A = 1\mu C$ is at $A(0,0,1)$ & $Q_B = -1\mu C$ is at $B(0,0,-1)$. Find E_r, E_θ & E_ϕ at $P(1,2,3)$

$$\vec{r}_{AP} = \hat{a}_x + 2\hat{a}_y + 2\hat{a}_z$$

$$\vec{r}_{BP} = \hat{a}_x + 2\hat{a}_y + 4\hat{a}_z$$

$$\vec{E} = \frac{Q_A}{4\pi\varepsilon_0|\vec{r}_{AP}|^2}\hat{r}_{AP} + \frac{Q_B}{4\pi\varepsilon_0|\vec{r}_{BP}|^2}\cdot\hat{r}_{BP}$$

$$= \frac{10^{-6}}{4\pi\varepsilon_0\times3^2}\cdot\frac{\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z}{3} - \frac{10^{-6}}{4\pi\varepsilon_0\times21}\times\frac{\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z}{\sqrt{21}}$$

$$= 239.7\hat{a}_x + \underline{479.4}\hat{a}_y + 292.4\hat{a}_z \text{ V/m}$$