MODULE -3 - Part-B

The Steady Magnetic Field

Books Referred:

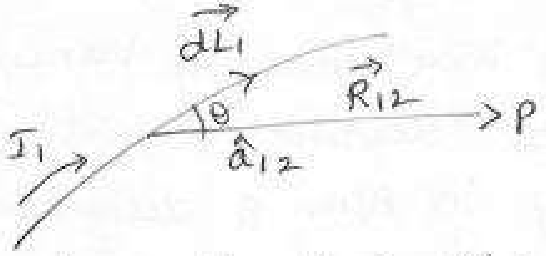
1) "Electromagnetics with Applications" John Krauss & Daniel A Fleisch, Mc-Graw Hill, 5th edition, 1999.

2) "Engineering Electromagnetics", William H Hayt Jr & John A Buck, Tata Mc-Graw-Hill 7th edition, 2006.

Topics Covered:

Biot-Savart law
Ampere's circuital law
Curl,
Stoke's theorem
Magnetic flux & flux density
Scalar & Vector magnetic potentials.
Numericals.

Biot - Savart's Law



It states that the magnetic field intensity due to the current $I\vec{dl}$ at any point P is

(a) directly proportional to current element.

(b) Directly proportional to sine of the $\angle^{le}$ b/w the current element & line joining current element at P.

(c) inversely proportional to square of distance b/w P & current element.

$$\vec{dH} \propto \frac{I\vec{dL}\sin\theta}{r^2}$$

$$\vec{dH} = \frac{I\vec{dl}\times\hat{a}_R}{4\pi R^2} = \frac{I\vec{dl}\times\vec{R}}{4\pi R^3} \quad A/m.$$

$$\boxed{\vec{H} = \oint \frac{I\vec{dl}\times\hat{a}_R}{4\pi R^2}}$$

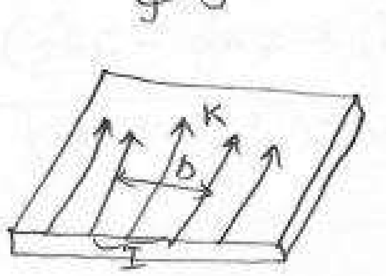
the Continuity eqn, $\nabla\cdot\vec{J} = -\frac{\delta\rho_v}{\delta t}$

The charge density is not a function of time

$$\nabla\cdot\vec{J} = 0$$

or by applying divergence theorem,

$$\oint \vec{J}\cdot\vec{ds} = 0$$



P.T.O

The Biot-Savart law also be expressed in terms of current density $\vec{J}$ & surface current density $\vec{K}$.

Surface current flows in a sheet of vanishingly small thickness, & the current density $\vec{J}$ in A/m^2. Surface current density in A/m & designated by $\vec{K}$.

If the surface current density is uniform, the total current, $\underline{I = Kb}$.

The width b is measured $\perp^r$ to the direction in which the current is flowing.

$$I = \int K\, dN$$

where dN is a differential element of the path across which current is flowing.

$$I\vec{dL} = \vec{K} ds = \vec{J} dv$$

$$\therefore \quad H = \int_S \frac{\vec{K} \times \hat{a}_R ds}{4\pi R^2}$$

$$H = \int_{Vol} \frac{\vec{J} \times \hat{a}_R dv}{4\pi R^2}$$

1) given the following values for P_1, P_2 & I_1, ΔL, calculate ΔH_2.

(a) $P_1 (0,0,2)$, $P_2 (4,2,0)$, $2\pi \hat{a}_z$ μA/m.

$$\Delta\vec{H_2} = \frac{I\vec{dL} \times \hat{a}_{R12}}{4\pi R^2}$$

$$= \frac{2\pi \hat{a}_z + (4\hat{a}_x + 2\hat{a}_y - 2\hat{a}_z) \times 10^{-6}}{4\pi (4^2 + 2^2 + 2^2)}$$

$$= \frac{(4\hat{a}_y - 2\hat{a}_x)}{2 \times 24 \times \sqrt{24}} = -8.51\hat{a}_x + 17.01\hat{a}_y \text{ nA/m}$$

(b) $P_1(0,2,0)$, $P_2(4,2,3)$, $2\pi\hat{a}_z$ μA-m

$$\Delta\vec{H_2} = \frac{I\,\vec{dl} \times \hat{a}_{R_{12}}}{4\pi R^2}$$

$$= \frac{2\pi\hat{a}_z \times (4\hat{a}_x + 0 + 3\hat{a}_z)}{4\pi(16+9)^{3/2}} \times 10^{-6}$$

$$= \frac{4\hat{a}_y}{2(25)^{3/2}} = 16\hat{a}_y \text{ nA/m}$$

(c) $P_1(1,2,3)$, $P_2(-3,-1,2)$, $2\pi(-\hat{a}_x + \hat{a}_y + 2\hat{a}_z)$ μA-m

$$\Delta\vec{H_2} = \frac{2\pi(-\hat{a}_x + \hat{a}_y + 2\hat{a}_z) \times (-4\hat{a}_x - 3\hat{a}_y - \hat{a}_z)}{4\pi(16+9+1)^{3/2}}$$

$$= \frac{(3\hat{a}_z - \hat{a}_y + 4\hat{a}_z - \hat{a}_x - 8\hat{a}_y + 6\hat{a}_x)}{2(26)^{3/2}} \times 10^{-6}$$

$$= 18.9\hat{a}_x - 33.9\hat{a}_y + 26.4\hat{a}_z \text{ nA/m}$$

2) Derive an expression for a magnetic field intensity at any point due to an infinite long current carrying Conductor.

$$\vec{R}_{12} = \vec{r} - \vec{r}' = \rho\hat{a}_\rho - z\hat{a}_z$$

$$\hat{a}_{R12} = \frac{\rho\hat{a}_\rho - z\hat{a}_z}{\sqrt{\rho^2+z^2}}$$

$$\vec{dL} = dz\,\hat{a}_z$$

$$\vec{dH_2} = \frac{I\,dz\,\hat{a}_z \times (\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi\,(\rho^2+z^2)^{3/2}}$$

$$\vec{H_2} = \int_{-\infty}^{\infty} \frac{I\,dz\,\hat{a}_z \times (\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi\,(\rho^2+z^2)^{3/2}}$$

$$= \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{\rho\,dz\,\hat{a}_\phi}{(\rho^2+z^2)^{3/2}}$$

$$\tan\theta = \frac{\rho}{z}$$

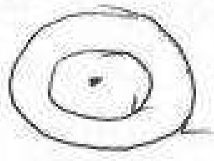
$$z = \rho\cot\theta \qquad dz = -\rho\,\mathrm{cosec}^2\theta\, d\theta$$

$$\vec{H_2} = \frac{I}{4\pi}\int_{\pi}^{0} \frac{(-\rho\,\mathrm{cosec}^2\theta)\,d\theta\,\hat{a}_\phi}{(\rho^2+\rho^2\cot^2\theta)^{3/2}}$$

$$\vec{H_2} = \frac{-I}{4\pi\rho}\int_{\pi}^{0} \sin\theta\, d\theta\, \hat{a}_\phi$$

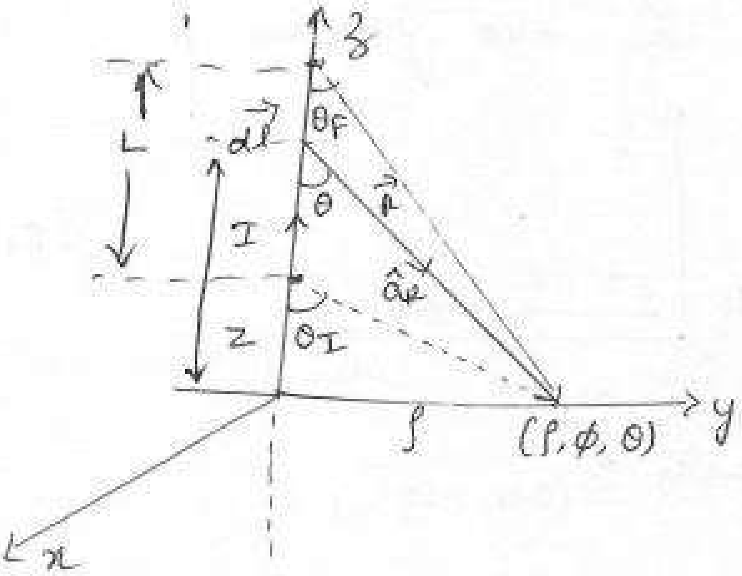
$$= \frac{-I}{4\pi\rho}\left(-\cos\theta\right)_{\pi}^{0}\hat{a}_\phi$$

$$\boxed{\vec{H_2} = \frac{I}{2\pi\rho}\hat{a}_\phi} \text{ A/m}$$



the Streamlines are ce abt the filament

2) Find the magnetic field intensity at any point in the vicinity of a straight current filament of finite length.



$$\vec{dH} = \frac{I\vec{dl} \times \hat{a}_R}{4\pi R^2} = \frac{I\,dz\,\hat{a}_z + (\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi(\rho^2+z^2)^{3/2}}$$

$$\vec{dH} = \frac{I\,dz\,\rho\hat{a}_\phi}{4\pi(\rho^2+z^2)^{3/2}}$$

from fig, $\tan\theta = \frac{\rho}{z} \Rightarrow z = \rho\cot\theta$

$$dz = -\rho\,\mathrm{cosec}^2\theta\,d\theta$$

$$\vec{dH} = \frac{-I}{4\pi\rho}\sin\theta\,d\theta\,\hat{a}_\phi$$

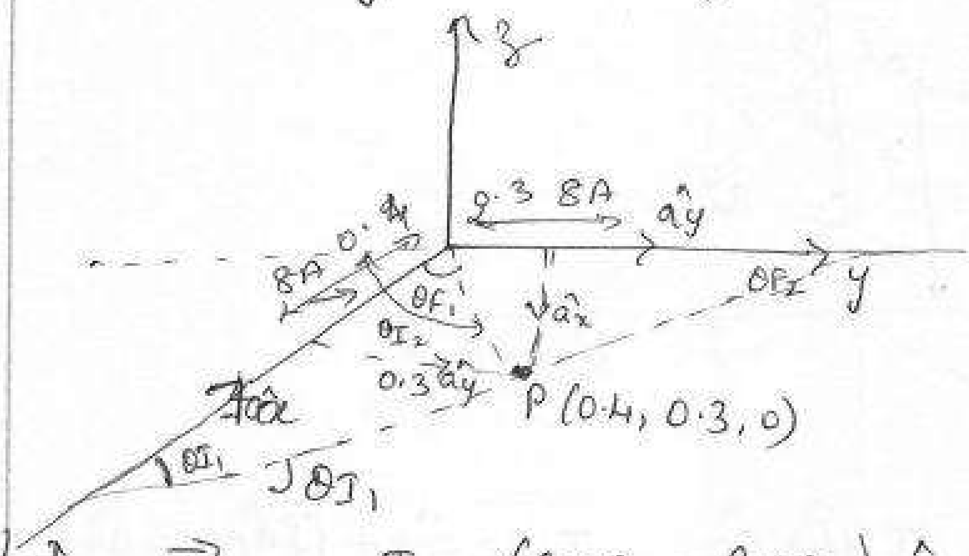
to find the total field,

$$\vec{H} = \frac{-I}{4\pi\rho}\int_{\theta_I}^{\theta_F}\sin\theta\,d\theta\,\hat{a}_\phi$$

$$= \frac{-I}{4\pi\rho}\left[-\cos\theta\right]_{\theta_I}^{\theta_F}$$

$$\vec{H} = \frac{I}{4\pi\rho}\left[\cos\theta_F - \cos\theta_I\right]\hat{a}_\phi \quad A/m$$

3) Find $\vec{H}$ at $P(0.4, 0.3, 0)$ in the field of an 8A filamentary current directed inward from infinity to origin on +ve x-axis, & then outward to ∞ along the +ve y-axis.



$$\theta_{F1} = \tan^{-1}\left(\frac{0.3}{0.4}\right) = \underline{36.87^\circ}$$

$$\vec{H}_{2(x)} = \frac{I}{4\pi\rho}(\cos\theta_F - \cos\theta_I)\,\hat{a}_\phi$$

$$\vec{H}_{2(x)} = \frac{8}{4\pi \times 0.3}(\cos 36.87 - \cos 180^\circ)\,\hat{a}_\phi = 3.82\,\hat{a}_\phi$$

$$= \frac{8}{4\pi \times 0.3}(\cos 36.87 - \cos 180)(-\hat{a}_x \times \hat{a}_y)$$

$$= -\underline{3.82}\,\hat{a}_z$$

$$\vec{H}_{2(y)} = \frac{8}{4\pi \times 0.4}(-\cos(90+36.8) + \cos 0)\,\hat{a}_y \times \hat{a}_x$$

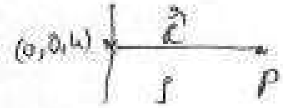
$$= -2.54\,\hat{a}_z$$

$$\vec{H} = \vec{H}_{(x)} + \vec{H}_{(y)} = -6.36\,\hat{a}_z \text{ A/m}$$

4) A current filament carrying 15A in the $\hat{a}_z$ direction lies along the entire z-axis. Find $\vec{H}$ in rectangular co-ordinates at (a) $P_A(\sqrt{20}, 0, 4)$.

Soln:- $|\vec{R}| = \rho = \sqrt{20}$ $\qquad \hat{a}_\phi = \vec{dl} \times \hat{a}_R$

$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{15}{2\pi\sqrt{20}}\hat{a}_\phi$ A/m



$= \frac{15}{2\pi\sqrt{20}}\hat{a}_z \times \hat{a}_x = 0.53\hat{a}_y$ A/m

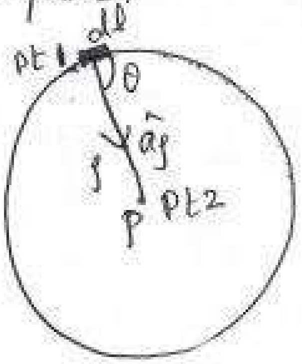
(b) $P_B(2,-4,4)$

$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_z \times \frac{(2\hat{a}_x - 4\hat{a}_y)}{\sqrt{20}}$

$= \frac{15\ \hat{a}_z \times (2\hat{a}_x - 4\hat{a}_y)}{2\pi\sqrt{20}\times\rho}$

$= \underline{\underline{0.476\hat{a}_x}} + 0.238\hat{a}_y$ A/m

5) find the magnetic field intensity at the centre of a circular conductor carrying a current of I amps.



Consider a current carrying conductor arranged in a circular form. $\vec{H}$ at the centre of the circular loop is to be obtained. The conductor carries the direct current I.

Consider a differential length $\vec{dl}$ at the point 1. The direction of $\vec{dl}$ at a pt 1 is tangential to the circular conductor at pt 1.

using the definition of cross product

$\vec{dl} \times \hat{a}_\rho = dl \sin\theta\, \hat{a}_N \qquad |\hat{a}_\rho| = 1$

where $\hat{a}_N \rightarrow$ unit vector normal to the plane containing $\vec{dl}$ & $\hat{a}_R$ i.e, normal to the plane in which the circular conductor is laying.

∴ the differential magnetic field dH at point P_2 is

$$\vec{dH} = \frac{I\vec{dl} \times \hat{a}_\rho}{4\pi\rho^2} = \frac{I\,dl\sin\theta\,\hat{a}_N}{4\pi\rho^2}$$

∴ the total MFI at point 2 due to the circular loop is $\vec{H} = \oint_l \vec{dH} = \oint \frac{I\,dl\sin\theta\,\hat{a}_N}{4\pi\rho^2} = \frac{I\sin\theta}{4\pi\rho^2}\oint dl\,\hat{a}_N$

But $\oint dl$ = circumference of the circle $= 2\pi\rho$ $[2\pi r]$

$$\therefore \vec{H} = \frac{I\sin\theta\,\cancel{2\pi\rho}\,\hat{a}_N}{\cancel{4\pi\rho^2}} = \frac{I\sin\theta\,\hat{a}_N}{2\rho}$$

as $I\vec{dl}$ is tangential to the circle & ρ is the radius ∴ angle θ must be $90°$.

$$\boxed{\therefore \vec{H} = \frac{I\sin 90°}{2\rho}\hat{a}_N = \frac{I\,\hat{a}_N}{2\rho}\ \text{A/m}}$$ $\quad | \sin 90° = 1$

& if the circular loop is placed in xy plane ie, $z=0$ plane $\hat{a}_N = \hat{a}_z$

then $\boxed{\vec{H} = \frac{I}{2\rho}\hat{a}_z\ \text{A/m}}$

Now $\vec{B} = \mu_0\vec{H}$ for free space.

the flux density $\vec{B}$ at the centre of the circular conductor carrying direct current I placed in a free space is given by

$$\boxed{\vec{B} = \mu_0\frac{I}{2\rho}\hat{a}_z} \quad \text{Wb/m}^2.$$

* Ampere's circuital law 6 to 7 marks

Statement: The line integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

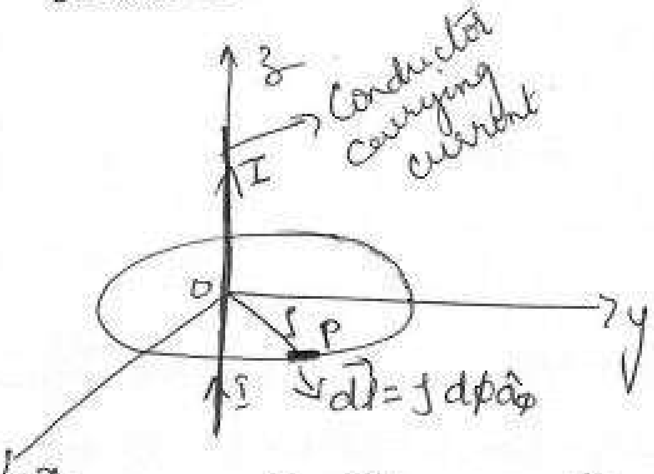
$$\boxed{\oint \vec{H}\cdot \vec{dl} = I}$$

when the current distribution is symmetrical law is useful

Proof:

Let us consider a long straight conductor carrying current 'I' along z-axis as shown.

Consider a closed path which encloses straight conductor.



Let us consider a small differential length $\vec{dL}$ at a point 'P'.

then we can write

$\vec{dL} = \rho\, d\phi\, \hat{a}_\phi$ (from fig)

at point 'P' the MFI is

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

Now $\vec{H}\cdot\vec{dl} = \frac{I}{2\pi\rho}\hat{a}_\phi \cdot \rho\, d\phi\, \hat{a}_\phi$ $\qquad \hat{a}_\phi\cdot\hat{a}_\phi = 1$

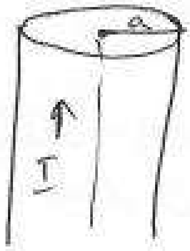
$$\vec{H}\cdot\vec{dl} = \frac{I}{2\pi}d\phi$$

Integrating over closed path

$$\oint \vec{H}\cdot\vec{dl} = \frac{I}{2\pi}\int_0^{2\pi} d\phi = \frac{I}{2\pi}\left[\phi\right]_0^{2\pi} = \frac{I}{2\pi}\times 2\pi$$

$$\boxed{\oint \vec{H}\cdot\vec{dl} = I}$$ proved.

1) Determine magnetic field intensity both inside & outside the conductor of radius 'a' m. The conductor carries a current of I amps using ampere's law.



for $\rho < a$

$$J = \frac{I}{\pi a^2}$$

$$\vec{H}\, 2\pi\rho = \pi\rho^2 \frac{I}{\pi a^2}$$

$$\vec{H} = \frac{I\rho}{2\pi a^2}\hat{a}_\phi$$

for $\rho > a$

$$\oint \vec{H}\cdot \vec{dl} = I$$

$$\int_0^{2\pi} H_\phi \cdot \rho\, d\phi = H_\phi\, 2\pi\rho \Rightarrow H_\phi\, 2\pi\rho = I$$

$$\boxed{H_\phi = \frac{I}{2\pi\rho}}$$

$$\boxed{\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi} \quad A/m$$

2) A Co-axial cable with radius of inner conductor 'a' inner radius of outer conductor 'b' & outer radius of c carries a current I at inner conductor & I in the outer conductor. Determine & sketch various of mag against (i) $\rho < a$ (ii) $a < \rho < b$ (iii) $b < \rho < c$ (iv) $\rho > c$

Soln:-

case (i) $\rho \leq a$

from Ampere's circuital law

$$\oint \vec{H}\cdot\vec{dl} = I_{enc} \Rightarrow \int H_\phi\, d\phi\, \rho\, d\phi\, \hat{a}_\phi = \frac{I\rho^2}{a^2}$$

$$\int_0^{2\pi} H_\phi \rho \, d\phi = \frac{I\rho^2}{a^2}$$

$$H_\phi \rho (\phi)_0^{2\pi} = \frac{I\rho^2}{a^2}$$

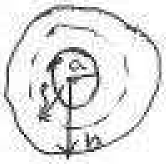
$$H_\phi = \frac{I\rho}{2\pi a^2}$$

MFI & MFD inside inner conductor

$$\boxed{\vec{H} = \frac{I\rho}{2\pi a^2}\hat{a}_\phi \ \ A/m} \quad (1) \quad \& \quad \boxed{\vec{B} = \frac{\mu I\rho}{2\pi a^2}\hat{a}_\phi \ \ wb/m^2} \quad (2)$$

Case (ii) $a < \rho < b$

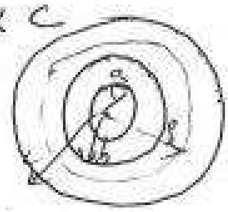
consider a closed path having radius $a < \rho < b$ this is similar to infinitely long filament placed along z-axis Hence



$$\boxed{\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi \ \ A/m \quad (3) \quad \& \quad \vec{B} = \frac{\mu I}{2\pi\rho}\hat{a}_\phi \ \ wb/m^2 \quad (4)}$$

Case (iii) $b < \rho < c$

Consider a closed path having radius $b < \rho < c$



the total current $-I$ is flowing through the cross-section $\pi(c^2-b^2)$ the closed path encloses the cross section $\pi(\rho^2-b^2)$ the current enclosed by closed path is

$$I_{encl} = I + \frac{\pi(\rho^2-b^2)}{\pi(c^2-b^2)}(-I)$$

$$= I - I\frac{(\rho^2-b^2)}{(c^2-b^2)} = \frac{I(c^2-b^2-\rho^2+b^2)}{(c^2-b^2)}$$

$$= \frac{I(c^2-\rho^2)}{(c^2-b^2)}$$

from Amperes circuital law

$$\oint \vec{H}\cdot\vec{dl} = I_{encl}$$

$$H_\phi \rho \int_0^{2\pi} d\phi = I\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$$H_\phi = \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$\vec{H} = H_\phi \hat{a}_\phi$

$\vec{dl} = \rho\, d\phi\, \hat{a}_\phi$

$$\vec{H} = \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)\hat{a}_\phi \quad A/m$$

$$\vec{B} = \frac{\mu I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)\hat{a}_\phi \quad Wb/m^2$$

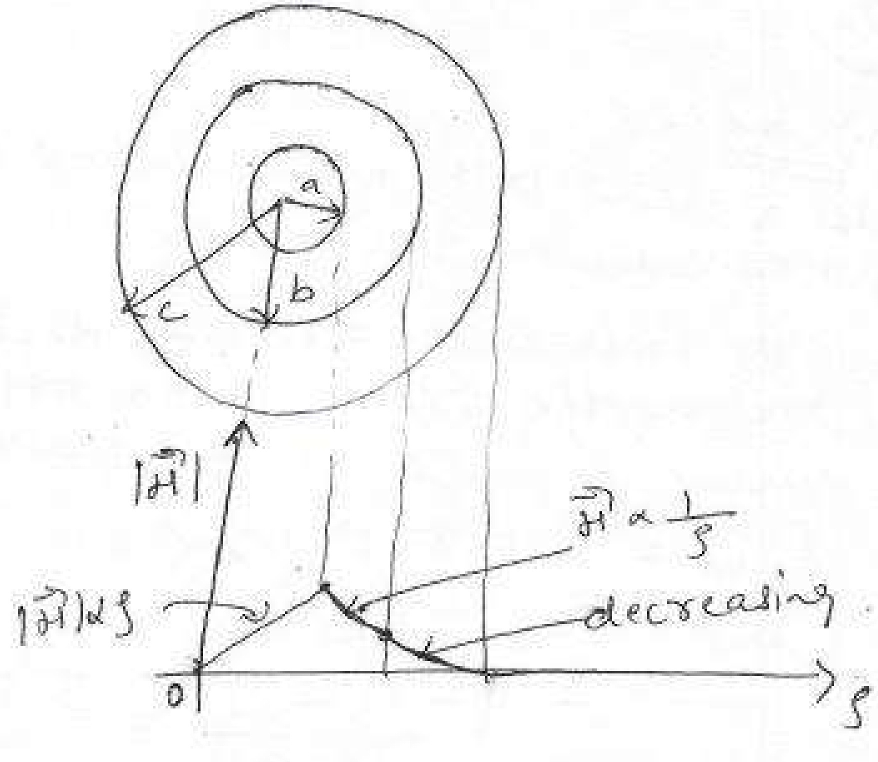
case (iv) $\rho > c$

Consider a closed path having radius $\rho > c$
total current enclosed is

$$I_{encl} = I - I = 0$$

$$\therefore \boxed{\vec{H} = \vec{B} = 0}$$

Plot



CURL

Curl of any vector point function gives the measure of the angular velocity at any point of the vector field.

Any motion in which the curl of the velocity vector is zero is said to be Irrotational otherwise Rotational.

Curl

Ampere's law in integral form,

$$\oint_l \vec{H}\cdot\vec{dl} = I$$

$\nabla \times \vec{A} = 0$ → Irrotational

Let ΔI be the current enclosed by closed path.

$$\oint_l \vec{H}\cdot\vec{dl} = \Delta I$$

Dividing both sides by ΔS & $\Delta S \to 0$

$$\lim_{\Delta S\to 0} \frac{\oint \vec{H}\cdot\vec{dl}}{\Delta S} = \lim_{\Delta S\to 0} \frac{\Delta I}{\Delta S}$$

The mathematical form (by defn)

$$\boxed{\text{Curl } \vec{H} = \nabla\times\vec{H} = \lim_{\Delta S\to 0} \frac{\oint_l \vec{H}\cdot\vec{dl}}{\Delta S}}$$

(it is line integral / unit area ← curl).

Hence $\nabla\times\vec{H} = \lim_{\Delta S\to 0}\frac{\Delta I}{\Delta S} = \vec{J}$

$\boxed{\oint \vec{H}\cdot\vec{dl} = I}$

$\boxed{\nabla\times\vec{H} = \vec{J}}$ → 2nd maxwell's eqn for non-time varying fields

$\oint \vec{E}\cdot\vec{dl} = 0$ or $\boxed{\nabla\times\vec{E} = 0}$ → 3rd Maxwells eqn " "

Point form.

Maxwells 2nd eqn is also known as Point form of ampere's law

Cartesian Coordinate form

$$\text{curl } \vec{H} = \nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ H_x & H_y & H_z \end{vmatrix} = \vec{J}$$

cylindrical form

$$\nabla \times \vec{H} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \delta/\delta\rho & \delta/\delta\phi & \delta/\delta z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix} = \vec{J}$$

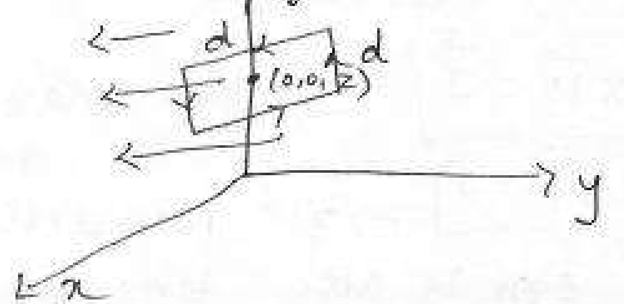
Spherical form

$$\nabla \times \vec{H} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\, \hat{a}_\phi \\ \delta/\delta r & \delta/\delta\theta & \delta/\delta\phi \\ H_r & rH_\theta & r\sin\theta H_\phi \end{vmatrix} = \vec{J}$$

Problems

1) Given $\vec{H} = 0.2 z^2 \hat{a}_x$ for $z > 0$ & $\vec{H} = 0$ elsewhere as shown. Calculate $\oint \vec{H} \cdot \vec{dl}$ about a square path wire with side d, centered at $(0, 0, z)$ in the $y=0$ Plane where $z_1 > d/2$, & $\nabla \times \vec{H}$.

soln:-



$$\oint \vec{H}\cdot \vec{dl} = 0.2\left(z_1+\frac{1}{2}d\right)^2 d + 0 - 0.2\left(z_1-\frac{1}{2}d\right)^2 d + 0$$

$$= \underline{0.4 z_1 d^2} \text{ Amps}$$

$$(\nabla \times \vec{H})_y = \lim_{d\to 0} \frac{\oint \vec{H}\cdot\vec{dl}}{d^2} = \lim_{d\to 0}\frac{0.4 z_1 d^2}{d^2} = \underline{0.4 z_1}$$

or

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ 0.2 z_1^2 & 0 & 0 \end{vmatrix} = \underline{0.4 z_1 \hat{a}_y} \text{ A/m}^2$$

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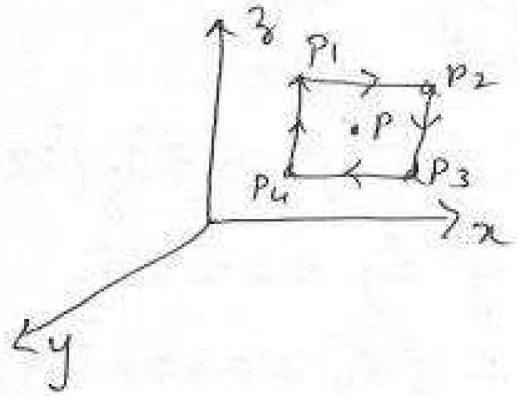
2) (a) Evaluate the closed line integral of $\vec{H}$ about the rectangular path $P_1(2,3,4)$ to $P_2(4,3,4)$ to $P_3(4,3,1)$ to $P_4(2,3,1)$ to P_1.

<u>Given</u>: $\vec{H} = 3z\hat{a}_x - 2x^3\hat{a}_z$ A/m



$$\oint \vec{H}\cdot\vec{dl} = \oint (3z\hat{a}_x - 2x^3\hat{a}_z)\cdot\vec{dl}$$

$$= 3z\,dx\big|_{P_1P_2} - (-2x^3)dz\big|_{P_2P_3} - dx\big|3z_{P_3P_4} + (-2x^3)dz\big|_{P_4P_1}$$

$$= 3(4)(4-2) + 2\,4^3(4-1) - 3(1)(4-2) + (-2)(2)^3(4-1)$$

$$= \underline{354\text{A}}$$

(b) Determine the quotient of closed line integral & the area enclosed by the path as an approximation to $(\nabla\times\vec{H})_y$.

<u>soln</u>:

$$(\nabla\times\vec{H})_y = \lim_{d\to 0}\frac{\oint \vec{H}\cdot\vec{dl}}{dx\,dz} = \frac{354}{2\times 3} = \underline{59 \text{ A/m}^2}$$

(c) Determine $(\nabla \times \vec{H})_y$ at the center of the area.

at Center $P(3, 3, 2.5)$

$$(\nabla \times \vec{H})_y = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ 3z & 0 & -2x^3 \end{vmatrix}$$

$$= -\hat{a}_y(-6x - 3) = 6x^2 + 3$$

$$(\nabla \times \vec{H})_P = 6 \times 3^2 + 3 = \underline{\underline{57 A/m^2}}$$

3) Calculate the value of vector current density

(a) in Rectangular Co-ordinates at $P_A(2, 3, 4)$

if $\vec{H} = x^2 z\, \hat{a}_y - y^2 x\, \hat{a}_z$

$$\vec{J} = \nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ 0 & x^2 z & y^2 x \end{vmatrix}$$

$$= \hat{a}_x\left[-x(2y) - x^2\right] - \hat{a}_y \cdot (-y^2 - 0) + \hat{a}_z(2xz - 0)$$

$$= -(2xy + x^2)\hat{a}_x + y^2 \hat{a}_y + 2xz\,\hat{a}_z$$

$$\vec{J}_{PA} = -(2 \times 2 \times 3 + 2^2)\hat{a}_x + 3^2 \hat{a}_y + 2 \times 2 \times 4\, \hat{a}_z$$

$$= \underline{\underline{-16\hat{a}_x + 9\hat{a}_y + 16\hat{a}_z}}\ A/m^2$$

(b) $\vec{H} = \frac{2}{\rho} \cos(0.2\phi)\, \hat{a}_\rho$ at $P_B(1.5, 90°, 0.5)$

$$\therefore \vec{J} = \nabla \times \vec{H} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \delta/\delta \rho & \delta/\delta \phi & \delta/\delta z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \delta/\delta\rho & \delta/\delta\phi & \delta/\delta z \\ \frac{2}{\rho}\cos(0\cdot 2\phi) & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\hat{a}_z\left(-\frac{2}{\rho}(-\sin 0.2\phi)\times 0.2\right)\right]$$

$$\vec{J}_P = \frac{0.4}{(1.5)^2}\sin(0.2(90^\circ))\,\hat{a}_z$$

$$= \underline{\underline{0.05\hat{a}_z}}\ A/m^2$$

(c) $\vec{H} = \frac{1}{\sin\theta}\hat{a}_\theta$ at $P_c(2, 30^\circ, 20^\circ)$.

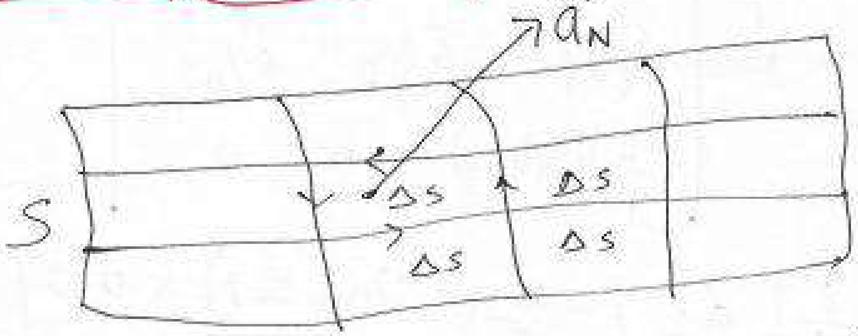
$$\vec{J} = \nabla\times\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \delta/\delta r & \delta/\delta\theta & \delta/\delta\phi \\ H_r & rH_\theta & r\sin\theta H_\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \delta/\delta r & \delta/\delta\theta & \delta/\delta\phi \\ 0 & \frac{r}{\sin\theta} & 0 \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[r\sin\theta\,\hat{a}_\phi\left(\frac{1}{\sin\theta}\right)\right] = \frac{1}{r\sin\theta}\hat{a}_\phi$$

$$= \frac{1}{2\times\sin 30^\circ}\hat{a}_\phi = \frac{1}{2\times\frac{1}{2}}\hat{a}_\phi = \underline{\underline{\hat{a}_\phi}}\ A/m^2$$

STOKES THEOREM



Consider the surface S shown above.
It is broken into incremental surface of area
ΔS.
Applying the definition of curl to one of these incremental surfaces,

$$\frac{\oint \vec{H} \cdot d\vec{l}_{\Delta S}}{\Delta S} = (\nabla \times \vec{H})_N$$

$d\vec{l}_{\Delta S}$ indicates the closed path, is the perimeter of an incremental area ΔS.

$$\frac{\oint \vec{H} \cdot d\vec{l}_{\Delta S}}{\Delta S} = (\nabla \times \vec{H}) \cdot \hat{a}_N$$

OR

$$\oint \vec{H} \cdot d\vec{l}_{\Delta S} = (\nabla \times \vec{H}) \cdot \hat{a}_N \cdot \Delta S = (\nabla \times \vec{H}) \cdot \vec{\Delta S}$$

where $\hat{a}_N \rightarrow$ unit vector.
in the direction of the rt-hand normal to ΔS.

The closed line integral for each ΔS, some cancellation will occur $\because$ every interior wall is covered once in each direction. The only boundaries on which cancellation cannot occur form the outside boundary, the path enclosing S.

$$\therefore \quad \boxed{\oint \vec{H} \cdot d\vec{l} = \int_S (\nabla \times \vec{H}) \cdot d\vec{s}}$$

where $\vec{dl}$ is taken on perimeter of S.
& above eqn is known as Stoke's theorem.

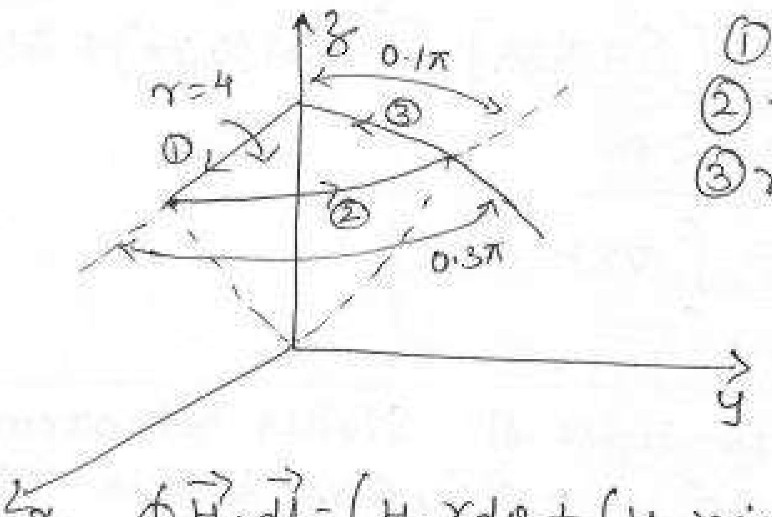
Ampere's law from Stokes theorem.

$$\oint_S \nabla\times\vec{H}\cdot\vec{ds} = \int_S \vec{J}\cdot\vec{ds} = I$$

$$\boxed{\oint_S \vec{H}\cdot\vec{dl} = I}$$

Problems:

1) Consider the portion of sphere as shown. The surface at $r=4$, $0\le\theta\le 0.1\pi$, $0\le\phi\le 0.3\pi$ & the closed path forming its perimeter is composed of three circular arcs.
given $\vec{H} = 6r\sin\phi\,\hat{a}_r + 18r\sin\theta\cos\phi\,\hat{a}_\phi$
Evaluate both sides of stoke's theorem.



① $r=4$, $0\le\theta\le 0.1\pi$, $\phi=0$
② $r=4$, $\theta=0.1\pi$, $0\le\phi\le 0.3\pi$
③ $r=4$, $0\le\theta\le 0.1\pi$, $\phi=0.3\pi$

$$\oint\vec{H}\cdot\vec{dl} = \int_{①} H_\theta\, r\,d\theta + \int_{②} H_\phi\, r\sin\theta\, d\phi + \int_{③} H_\theta\, r\,d\theta$$

Since $H_\theta = 0$ (given in $\vec{H}$)

$$\oint\vec{H}\cdot\vec{dl} = \int_0^{0.3\pi} 18r\sin\theta\cos\phi\; r\sin\theta\, d\phi$$

at $\theta = 0.1\pi$

$$= 18\times 4^2\sin^2(0.1\pi)\int_0^{0.3\pi}\cos\phi\, d\phi$$

$$= 18\times 4^2\times\sin^2(0.1\pi)\,[\sin(0.3\pi) - \sin 0]$$

$$= \underline{22.2A}$$

$$\nabla\times\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 6r\sin\phi & 0 & r\cdot r\,18\sin^2\theta\cos\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[\hat{a}_r\left(18r^2\,2\sin\theta\cos\theta\cos\phi\right) - r\hat{a}_\theta\left(18\times 2r\sin^2\theta\cos\phi - 6r\cos\phi\right)\right]$$

$$= 36\cos\theta\cos\phi\,\hat{a}_r - \left(36\sin\theta\cos\phi - \frac{6\cos\phi}{r\sin\theta}\right)\hat{a}_\theta$$

Since $\vec{ds}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$

$$\int_S \nabla\times\vec{H}\cdot\vec{ds}_r = \int_{\phi=0}^{0.3\pi}\int_{\theta=0}^{0.1\pi}(36\cos\theta\cos\phi)\,16\sin\theta\,d\theta\,d\phi$$

$$= 576\int_0^{0.3\pi}\cos\phi\,d\phi\int_0^{0.1\pi}\frac{\sin^2\theta}{2}\,d\theta.$$

$$= \frac{576}{2}\left[\sin\phi\right]_0^{0.3\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{0.1\pi}$$

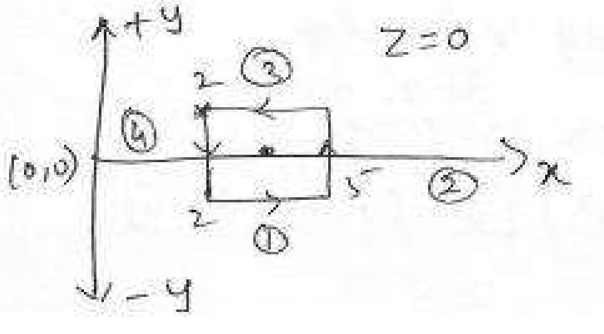
$$= \frac{288}{2}\left[\sin 0.3\pi\right]\left[-\cos(0.2\pi)+\cos 0\right]$$

$$= \underline{22.2\ A}$$

$$\boxed{\oint\vec{H}\cdot\vec{dl} = \int_S \nabla\times\vec{H}\cdot\vec{ds}}$$

---

2) Evaluate both sides of Stokes theorem for the field $\vec{H} = 6xy\,\hat{a}_x - 3y^2\,\hat{a}_y$ A/m & the rectangular path around the region, $2\le x\le 5$, $-1\le y\le 1$, $z=0$. Let the +ve direction of $\vec{ds}$ be $\hat{a}_z$.



$$LHS = \oint \vec{H}\cdot\vec{dl} = \int_{①} H_x\,dx + \int_{②} H_y\,dy - \int_{③} H_x\,dx - \int_{④} H_y\,dy$$

$$= \int_2^5 6xy\,dx \Big|_{y=-1} + \int_{-1}^{+1} -3y^2\,dy \Big|_{x=5} - \int_2^5 6xy\,dx \Big|_{y=1} - \int_{-1}^{+1} -3y^2\,dy \Big|_{x=2}$$

$$= \frac{6}{2}x^2\Big]_2^5 (-1) + (-3)\left[\frac{y^3}{\cancel{3}}\right]_{-1}^{+1} - \frac{6}{2}(x^2)_2^5(1) + 3\left.\frac{y^3}{3}\right]_{-1}^{+1}$$

$$= -3(25-4) - 3(25-4)$$

$$= \underline{\underline{-126A}}$$

$$\nabla\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ 6xy & -3y^2 & 0 \end{vmatrix}$$

$$= \hat{a}_x(0-0) - \hat{a}_y(0) + \hat{a}_z(-6x)$$

$$= \underline{\underline{-6x\hat{a}_z}}$$

$$\int \nabla\times\vec{H}\cdot\vec{ds} = -\int_{x=2}^{5}\int_{y=-1}^{+1} 6x\,dx\,dy$$

$$= -6\frac{x^2}{2}\Big]_2^5 \left[y\right]_{-1}^{+1} = -3(25-4)(1-(-1))$$

$$= \underline{\underline{-126A}}$$

$$\therefore \oint \vec{H}\cdot\vec{dl} = \int_s \nabla\times\vec{H}\cdot\vec{ds} = \underline{\underline{-126A}}$$

Magnetic flux & Magnetic flux density

In free space, the magnetic flux density $\vec{B}$ is,

$$\boxed{\vec{B} = \mu_0 \vec{H}} \quad Wb/m^2 \text{ or Tesla (T)}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m.

μ_0 = permeability in free space.

$H \longleftrightarrow E$
$B \longleftrightarrow D$

The magnetic flux, ϕ passing through any designated area,

$$\boxed{\Phi = \int_S \vec{B} \cdot \vec{ds} \quad Wb}$$

$\Phi = \psi$

The charge Q is the source of electric flux & these lines begin & terminate on +ve & -ve charges respectively.

No such source has ever been discovered for the lines of magnetic flux.

The magnetic flux lines are closed & do not terminate on a magnetic charge.

∴ Gauss's law for magnetic field is,

$$\boxed{\oint_S \vec{B} \cdot \vec{ds} = 0} \quad \text{(in integral form)}.$$

& application of divergence theorem,

i.e, $$\underset{\Delta v \to 0}{Lt} \frac{\oint_S \vec{B} \cdot \vec{ds}}{\Delta v} = 0$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Maxwell's 4th eqn for steady magnetic fields in point forms (Point form of Gauss law for steady magnetic fields).

Maxwells equation in point form for static electric & steady magnetic fields.

$$\boxed{\begin{aligned} \nabla\cdot\vec{D} &= \rho_v \\ \nabla\times\vec{E} &= 0 \\ \nabla\times\vec{H} &= \vec{J} \\ \nabla\cdot\vec{B} &= 0 \end{aligned}}$$

integral form

$$\boxed{\begin{aligned} &\oint \vec{D}\cdot d\vec{s} = Q = \int_{vol} \rho_v\, dv \\ &\oint \vec{E}\cdot d\vec{l} = 0 \\ &\oint \vec{H}\cdot d\vec{l} = I = \int_s \vec{J}\cdot d\vec{s} \\ &\oint_s \vec{B}\cdot d\vec{s} = 0 \end{aligned}}$$

Point form

$$\boxed{\begin{aligned} \vec{D} &= \varepsilon_0 \vec{E} \\ \vec{B} &= \mu_0 \vec{H} \end{aligned}}$$

Problems:

1) Find the magnetic flux crossing a Co-axial Cable of $\rho = a$ to $\rho = b$ & also $z = 0$ to $z = d$.

soln:- From amperes law,

$$a < \rho < b$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$B_\phi = \mu_0 H_\phi = \frac{\mu_0 I}{2\pi\rho}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\hat{a}_\phi$$

$$\phi = \int_s \vec{B}\cdot d\vec{s} = \int_{z=0}^{d}\int_{\rho=a}^{b} \frac{\mu_0 I}{2\pi\rho}\hat{a}_\phi \cdot d\rho\, dz\, \hat{a}_\phi$$

$$\boxed{\Phi = \frac{\mu_0 I d}{2\pi} \ln(b/a) \quad wb}$$

2) A solid conductor of circular cross section is made of a homogenous non-magnetic material. If radius $a = 1mm$, the conductor axis lie on Z-axis. & the total current $\hat{a}_z$ direction is 20A.

Find (a) H_ϕ on $\rho = 0.5mm$

for $\rho < a$

$0.5 < 1$

from ampere's law

$$\oint \vec{H} \cdot \vec{dl} = I$$

$$H_\phi 2\pi\rho = \frac{I}{\pi a^2} \cdot \pi\rho^2$$

$$H_\phi = \frac{I\rho}{2\pi a^2} = \frac{20 \times 0.5 \times 10^{-3}}{2\pi \times 10^{-6}} = 1592 \, A/m$$

(b) B_ϕ at $\rho = 0.8mm$

$$B_\phi = \mu_0 H_\phi = 4\pi \times 10^{-7} \times \frac{20 \times 0.8 \times 10^{-3}}{2\pi \times 10^{-6}} = 3.2mT = 3.2 mWb/m^2$$

(c) total magnetic flux/unit length inside the conductor.

$$\Phi = \int_s \vec{B} \cdot \vec{ds} = \int_0^l \int_0^a \frac{\mu_0 I \rho}{2\pi a^2} d\rho \, dz$$

$$= \frac{\mu_0 I}{2\pi a^2} \left.\frac{\rho^2}{2}\right]_0^a l = \frac{\mu_0 I l}{4\pi}$$

$$\Phi / \text{length} = \frac{\mu_0 I}{4\pi} = \frac{20 \times 4\pi \times 10^{-7}}{4\pi} = 2 \mu Wb/m$$

(d) total flux $\rho < 0.5mm$

Let $l = 1m$

$$\Phi = \frac{\mu_0 I l}{2\pi a^2} \left.\frac{\rho^2}{2}\right]_0^{0.5 \times 10^{-3}} = \frac{4\pi \times 10^{-7} \times 20 \times 1 \, (0.5)^2 \times 10^{-6}}{4\pi \times 1 \times 10^{-6}} = 0.5 \mu Wb$$

(e) the total magnetic flux outside the conductor.

$$\Phi = \int_s \vec{B} \cdot \vec{ds} = \int_s \frac{I}{2\pi\rho} d\rho \, dz = \int_0^l \int_a^\infty \frac{I}{2\pi\rho} d\rho \, dz$$

$$= \frac{Il}{2\pi} \ln\infty = \infty$$

Scalar Magnetic potential

Similar to Electromagnetic potential V, if magnetic Scalar potential exists, which can be found from the current distribution, if designated as V_m.

$$\Rightarrow \vec{H} = -\nabla V_m .$$

the definition must not conflict with any of results of magnetic fields.

$$\nabla \times \vec{H} = \vec{J} = \nabla \times (-\nabla V_m)$$

$$\vec{J} = 0$$

∵ curl of gradient of any Scalar is zero.

Hence, $\boxed{\vec{H} = -\nabla V_m \quad \text{for } (\vec{J} = 0)}$

$\vec{H} \rightarrow$ is to be defined as gradient of Scalar magnetic potential then current density must be zero. throughout the region in which scalar magnetic Potential is also defined.

The Scalar magnetic potential also satisfies Laplace eqn:

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \mu_0 \vec{H} = 0$$

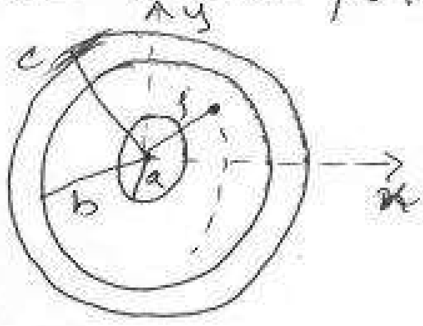
$$\nabla \cdot \mu_0 (-\nabla V_m) = 0$$

$$-\mu_0 \nabla^2 V_m = 0$$

$$\therefore \boxed{\nabla^2 V_m = 0 \quad \text{for } (\vec{J} = 0)}$$

V_m Satisfies laplace's equation.

One difference b/w V & V_m is V_m is not a single valued function of position.
But electric potential V is single valued.



Consider a cross-section of Co-axial line. $a < \rho < b$, $\vec{J} = 0$.

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

$$\therefore \vec{H} = -\nabla V_m .$$

$$\frac{I}{2\pi\rho}\hat{a}_\phi = -\frac{1}{\rho}\frac{\delta V_m}{\delta\phi}\hat{a}_\phi$$

$$\frac{\delta V_m}{\delta\phi} = -\frac{I}{2\pi} \quad \text{or} \quad V_m = -\frac{I}{2\pi}\phi$$

(where const of integn has been set to zero.

If $\phi = 0 \Rightarrow V_m = 0$.
& proceeding counter clkwise direction around the circle, V_m goes $-$ve.
at $\phi = 2\pi$, $\Rightarrow$ $V_m = -I$ (but at the same point $V_m = 0$ at $\phi = 0$).

the reason for multi value,

$$\nabla \times \vec{E} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = 0.$$

$\therefore V_{ab} = -\int_b^a \vec{E}\cdot d\vec{l}$ is independent of path.

But $\nabla \times \vec{H} = 0$ (whenever $\vec{J} = 0$).

& $\oint \vec{H}\cdot d\vec{l} = I$

Every time we make another complete loop around the path the result of the integration $\uparrow$es by I.
If no current is enclosed by that path, then single valued potential function may be defined.

in general, $\boxed{V_{m,ab} = -\int_b^a \vec{H}\cdot d\vec{l} \quad \text{(specified path)}}$

Vector Magnetic Potential

The vector magnetic potential may be used in regions where the current density is zero or non-zero.

This vector field is useful in studying radiation from antennas & from apertures & radiation leakage from transmission lines, waveguides & microwave ovens.

W.K.T $\nabla \cdot \vec{B} = 0$.

Divergence of curl of any vector field is zero.

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

$\vec{A} \rightarrow$ signifies Vector magnetic Potential.

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0} \nabla \times \vec{A}$$

&

$$\nabla \times \vec{H} = \vec{J} = \frac{1}{\mu_0} \nabla \times (\nabla \times \vec{A}) \quad \text{—} ①$$

$$(\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A})$$

the unit of $\vec{A}$ is Wb/m.

from Biot-Savart's law,

$$\vec{A} = \oint \frac{\mu_0 I \, d\vec{l}}{4\pi R}$$

the above equation signifies a direct current I flows along a filamentary conductor of which any differential length $d\vec{L}$ is distant R from the point at which $\vec{A}$ is to be found.

from equation ①

$$\frac{1}{\mu_0}\nabla\times(\nabla\times\vec{A}) = \vec{J}$$

$$\nabla\cdot(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{J}$$

for steady magnetic fields, $\nabla\cdot\vec{A} = 0$.

under static or dc conditions,

$$-\nabla^2\vec{A} = \mu_0\vec{J}$$

$$\boxed{\nabla^2\vec{A} = \frac{-\vec{J}}{1/\mu_0}} \quad \text{— (2)}$$

this eqn is similar to $\nabla^2 V = \frac{-\rho_v}{\varepsilon}$

Hence eqn (2) known as poisson's eqn for steady magnetic fields.

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\varepsilon}} \quad \text{— (3)}$$

eqn (2) & (3) are analogous if 2 eqns are mathematically identical.

i.e,

$$A \Longleftrightarrow V$$

$$\vec{J} \Longleftrightarrow \rho_v$$

$$\frac{1}{\mu} \Longleftrightarrow \varepsilon$$

absolute potential at any point at a distance of r mts, $V = \frac{Q}{4\pi\varepsilon R}$

if the region consists of continous charge distribution the above eqn.

$$V = \int_{vol} \frac{\rho_v dv}{4\pi\varepsilon R}.$$

By analogy,

$$\vec{A} = \int_{vol} \frac{\vec{J}\cdot dv}{4\pi\frac{1}{\mu}\cdot R}$$

$$= \int_l \int_s \frac{(\vec{J} \cdot \vec{ds})\, \vec{dl}}{4\pi \frac{1}{\mu} R}$$

But $\int_l \vec{J} \cdot \vec{ds} = I$

$$= \int_l \frac{I\, dl}{4\pi (\frac{1}{\mu}) R}.$$

$$\boxed{\vec{A} = \int_l \frac{\mu_0 I \vec{dl}}{4\pi R} \quad Wb/m}$$

$r \rightarrow$ distance b/w current element $\vec{dl}$ & point 'P'.

In terms of surface current density,

since $I\vec{dl} = \vec{K}\, ds$

$$\boxed{\vec{A} = \int_s \frac{\vec{K}\, ds}{4\pi \frac{1}{\mu} R}}$$

&

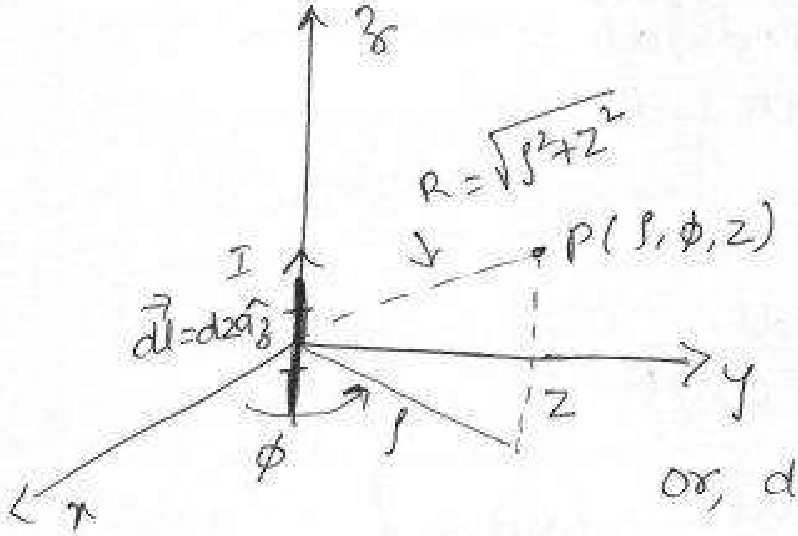
$$I\vec{dl} = \vec{J}\, dv = \vec{K}\, ds.$$

$$\boxed{\vec{A} = \int_{vol} \frac{\vec{J} \cdot dv}{4\pi \frac{1}{\mu} R}}$$

Problem:-

1) A finite length current carrying conductor is placed along Z-axis, as origin as center of conductor. Find differential vector magnetic potential at $P(\rho, \phi, z)$ and also from there differential magnetic field.

Soln:-



W.K.T

$$\vec{A} = \int_l \frac{\mu_0 I \vec{dl}}{4\pi R}$$

$$\vec{dA} = \frac{\mu_0 I dz \hat{a}_z}{4\pi \sqrt{\rho^2 + z^2}}$$

or, $dA_z = \dfrac{\mu_0 I dz}{4\pi\sqrt{\rho^2+z^2}}$, $dA_\phi = 0$ & $dA_\rho = 0$.

$\vec{dA}$ is in same direction of $I\vec{dl}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0}(\nabla \times \vec{A})$$

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$d\vec{H} = \frac{1}{\mu_0} \nabla \times d\vec{A}$$

$$= \frac{1}{\mu_0}\left(-\frac{\partial (dA_z)}{\partial \rho}\right)\hat{a}_\phi$$

$$\boxed{d\vec{H} = \frac{I dz}{4\pi} \frac{\rho}{(\rho^2+z^2)^{3/2}} \hat{a}_\phi}$$

Which is same as the value given by Biot - Savart's law.

4) given the potential field $V = (A\rho^4 + B\rho^{-4})\sin 4\phi$

S.T $\nabla^2 V = 0$

$$\nabla^2 V = \frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) + \frac{1}{\rho^2}\frac{\delta^2 V}{\delta\phi^2} + \frac{\delta^2 V}{\delta z^2}$$

$$\frac{\delta V}{\delta\rho} = (4A\rho^3 - 4B\rho^{-5})\sin 4\phi$$

$$\frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) = \frac{1}{\rho}\frac{\delta}{\delta\rho}\left[4A\rho^4 - 4B\rho^{-4}\right)\sin 4\phi$$

$$= \frac{1}{\rho}\left[16A\rho^3 + 16B\rho^{-5}\right)\sin 4\phi$$

$$= (16A\rho^2 + 16B\rho^{-6})\sin 4\phi \quad -①$$

$$\frac{\delta V}{\delta\phi} = (A\rho^4 + B\rho^{-4})\, 4\cos 4\phi$$

$\frac{d}{d}\cos = -\sin$

$\int \sin = -\cos\theta$

$$\frac{\delta^2 V}{\delta\phi^2} = (A\rho^4 + B\rho^{-4})\, 16(-\sin 4\phi)$$

$$= -16(A\rho^4 + B\rho^{-4})\sin 4\phi \quad -②$$

$$\frac{1}{\rho^2}\frac{\delta^2 V}{\delta\phi^2} = -16(A\rho^2 + B\rho^{-6})\sin 4\phi$$

① + ②

$$\nabla^2 V = [\cancel{16A\rho^2} + \cancel{16B\rho^{-6}}]\cancel{\sin 4\phi} + [\cancel{-16A\rho^2} - 16B\rho^{-6})\sin 4\phi$$

$16A\rho^2\sin 4\phi + 16B\rho^{-6}\sin 4\phi - 16A\rho^2\sin 4\phi - 16B\rho^{-6}\sin 4\phi$

$$\nabla^2 V = 0$$

5) Select A & B so that $V = 100V$ & magnitude of $|\vec{E}| = 500 \, V/m$ at $P(\rho=1, \phi = 22.5^\circ, z = 2)$

$$V_P = \left[A(1)^4 + B(1)^{-4}\right] \sin(4 \times 22.5^\circ)$$

$$100 = (A+B) \underbrace{\sin(90^\circ)}_{1}$$

$$\underline{\underline{A + B = 100}}$$

$$\vec{E} = -\nabla V$$

$$= -\left[\frac{\delta V}{\delta \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\delta V}{\delta \phi}\hat{a}_\phi + \frac{\delta V}{\delta z}\hat{a}_z\right].$$

$$= -\left[(4A\rho^3 - 4B\rho^{-5}) \sin 4\underset{22.5}{\phi}\, \hat{a}_\rho + \frac{4}{\rho}(A\rho^4 + B\rho^{-4}) \cos 4\phi\right.$$

$$\vec{E}_P = -4\left[A - B\right] \sin 90^\circ \hat{a}_\rho + 4(A+B) \cos 90^\circ \hat{a}_\phi$$

$$= -4(A-B)\hat{a}_\rho + 0$$

$\cos 90 = 0$
$\sin 0 = 1$

$$|\vec{E}_P| = 500 \Rightarrow 500 = -4(A-B)$$

$$\frac{500}{4} = 125 = \underline{\underline{B - A}}$$

$B - A = 125$
$A + B = 100$
$2B = 225$

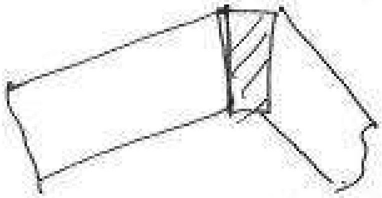
$$\boxed{B = \underline{\underline{112.5}}}$$

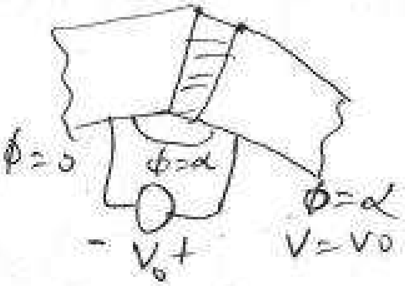
$B - A = 125$
$B - 125 = A$
$112.5 - 125 = A$

$$A = -12.5$$

3) Semi-infinite planes $\phi=0$ & $\phi=\alpha$ are separated by a very small insulating gap as shown. If $V(\phi=0)=0V$ $V(\phi=\alpha)=V_0$. Calculate voltage & $\vec{E}$ in the region b/w the planes.

Soln





$$\nabla^2 V = \frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right)+\frac{1}{\rho^2}\frac{\delta^2 V}{\delta\phi^2}+\frac{\delta^2 V}{\delta z^2}=0$$

$$\nabla^2 V = 0+\frac{1}{\rho^2}\frac{\delta^2 V}{\delta\phi^2}+0=0$$

$$\frac{\delta^2 V}{\delta\phi^2}=0$$

Integrating w.r.t ϕ

$$\frac{\delta V}{\delta\phi}=C_1$$

Inte again.

$$\boxed{V=C_1\phi+C_2}$$

apply B.C

$\phi=0,\quad V_\phi=0$

$\phi=\alpha,\quad V=V_0$

$$0=C_1(0)+C_2$$

$$\boxed{C_2=0}$$

$$V_0=C_1(\alpha)+C_2$$

$$V_0=C_1(\alpha)+0$$

$$\boxed{C_1=\frac{V_0}{\alpha}}$$

$$\therefore \boxed{V=\frac{V_0\phi}{\alpha}}\ V$$

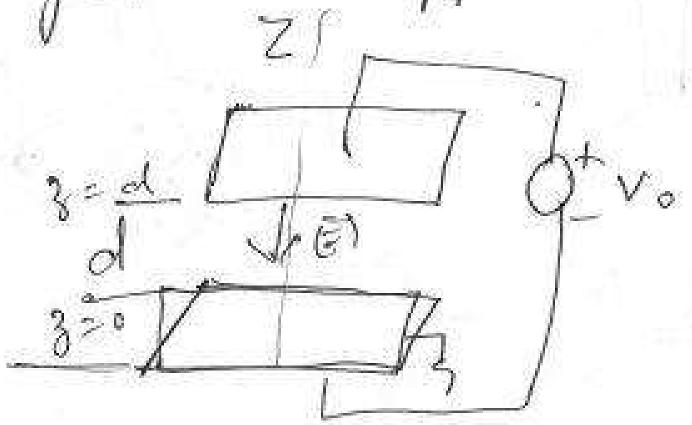
$$\vec{E}=-\nabla V$$

$$=-\left[\frac{\delta V}{\delta\rho}\hat{a}_\rho+\frac{1}{\rho}\frac{\delta V}{\delta\phi}\hat{a}_\phi+\frac{\delta V}{\delta z}\hat{a}_z\right]$$

$$=-\left[0+\frac{1}{\rho}\frac{\delta V}{\delta\phi}\hat{a}_\phi+0\right]$$

$$\boxed{\vec{E}=-\frac{1}{\rho}\frac{V_0}{\alpha}a_\phi}\ V/m$$

2) Applying Laplace's equation, obtain the expression for $\vec{E}$ in Cartesian Co-ordinates b/w 2 parallel planes separated by distance 'd' in z-direction having potential applied b/w them. Ie also find capacitance



Lap Eq $\nabla^2 V = \frac{\delta^2 V}{\delta x^2} + \frac{\delta^2 V}{\delta y^2} + \frac{\delta^2 V}{\delta z^2} = 0$.

Poten is a fn of z only.

$$\nabla^2 V = \frac{\delta^2 V}{\delta z^2} = 0.$$

Integrating w.r.t z twice.

$$\frac{\delta V}{\delta z} = C_1$$

$$V = C_1 z + C_2 \quad -(1)$$

$0 < z < d$

C_1, C_2 are integration const.

to find C_1, C_2 use B.C

at $Z=0$, $V=0$

$$0 = C_1 (0) + C_2 \Rightarrow \boxed{C_2 = 0}$$

at $Z=d$, $V = V_0$

$$V_0 = C_1(d) + C_2$$

$$V_0 = C_1(d) + 0$$

$$\boxed{C_1 = \frac{V_0}{d}}$$

$$\boxed{V = \frac{V_0}{d} z + 0} \quad -(2)$$

$$\vec{E} = -\nabla V = -\left[\frac{\delta V}{\delta x} a_x + \frac{\delta V}{\delta y} a_y + \frac{\delta V}{\delta z} \hat{a}_z\right]$$

$$= -\left[0 + 0 + \frac{V_0}{d}\hat{a}_z\right] \quad \Big| \frac{\delta}{\delta z}\left(\frac{V_0 z}{d}\right)$$

$$\boxed{\vec{E} = -\frac{V_0}{d}\hat{a}_z}$$

$\rho_s = |\vec{D}_N|$

$$\vec{D} = \varepsilon \vec{E} = -\varepsilon \frac{V_0}{d}\hat{a}_z$$

$C = Q/V_0$

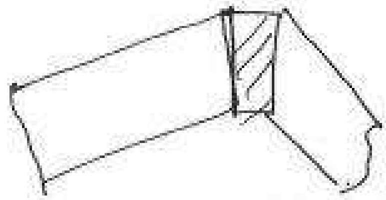
$$Q = \int_s \rho_s ds = \int_s \vec{D}_N ds = \int -\frac{\varepsilon V_0}{d} ds$$

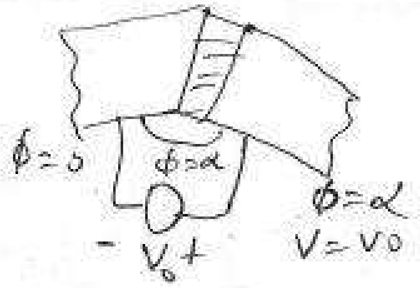
$$Q = -\frac{\varepsilon_0 V_0 S}{d}, \quad |Q| = \frac{\varepsilon V_0 S}{d}$$

$$C = \frac{|Q|}{V_0} = \frac{\frac{\varepsilon V_0 S}{d}}{V_0} = \boxed{\frac{\varepsilon_0 S}{d} = C}$$

3) Semi-infinite planes $\phi=0$ & $\phi=\alpha$ are separated by a very small insulating gap as shown. If $V(\phi=0)=0V$, $V(\phi=\alpha)=V_0$. Calculate Voltage & $\vec{E}$ in the region b/w the planes.

Soln





$$\nabla^2 V=\frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right)+\frac{1}{\rho^2}\frac{\delta^2 V}{\delta\phi^2}+\frac{\delta^2 V}{\delta z^2}=0$$

$$\nabla^2 V=0+\frac{1}{\rho^2}\frac{\delta^2 V}{\delta\phi^2}+0=0$$

$$\frac{\delta^2 V}{\delta\phi^2}=0$$

Integrating w.r.t ϕ

$$\frac{\delta V}{\delta\phi}=C_1$$

Inte again

$$\boxed{V=C_1\phi+C_2}$$

apply B.C

$\phi=0,\quad V=0$

$\phi=\alpha,\quad V=V_0$

$$0=C_1(0)+C_2$$

$$\boxed{C_2=0}$$

$$V_0=C_1(\alpha)+C_2$$

$$V_0=C_1(\alpha)+0$$

$$\boxed{C_1=\frac{V_0}{\alpha}}$$

$$\therefore\ \boxed{V=\frac{V_0\phi}{\alpha}}\ V$$

$$\vec{E}=-\nabla V$$

$$=-\left[\frac{\delta V}{\delta\rho}\hat{a}_\rho+\frac{1}{\rho}\frac{\delta V}{\delta\phi}\hat{a}_\phi+\frac{\delta V}{\delta z}\hat{a}_z\right]$$

$$=-\left[0+\frac{1}{\rho}\frac{\delta V}{\delta\phi}\hat{a}_\phi+0\right]$$

$$\boxed{\vec{E}=-\frac{1}{\rho}\frac{V_0}{\alpha}a_\phi}\ V/m$$

4) two radial Conducting planes $V = 50V$ at $\phi = 10°$

$V = 20V$ at $\phi = 30°$. $P(3,1,2)$ $|\vec{E}_P|$

$$\nabla^2 V = \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = 0 \Rightarrow \frac{\partial^2 V}{\partial \phi^2} = 0$$

Integrate once $\frac{\partial V}{\partial \phi} = C_1$

Integ one more time $\boxed{V = C_1\phi + C_2}$

$50 = C_1 10° + C_2$ — ①

$20 = C_1 30° + C_2$ — ②

① − ②

$50 - 20 = C_1 10° + \cancel{C_2} - C_1 30° - \cancel{C_2}$

$30 = C_1 10° - C_1 30°$

$30 = C_1[10° - 30°] \times \frac{\pi}{180°}$ to Convert into radian

$30 = C_1[-20°] \times \frac{\pi}{180}$ $\frac{20}{180}$

$C_1 = -\frac{30 \times 180}{\pi \times 20°}$ — keep calci in radian mode $\left(\frac{180°}{20°}\right) = 9$

$C_1 = -\frac{30 \times 9}{\pi} = -85.987$

$$\vec{E} = -\nabla V = -\frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi = -\frac{1 C_1}{\rho}\frac{\partial}{\partial \phi}(\phi)\cdot \hat{a}_\phi$$

$P(3,1,2)$
$\rho = \sqrt{x^2+y^2} = \sqrt{3^2+1^2}$
$= \sqrt{10}$
$= 3.16$

$$\vec{E} = -\frac{C_1 \hat{a}_\phi}{\rho} = -\frac{(-85.987)\hat{a}_\phi}{}$$

Calci in radian mode

$= \frac{85.987\hat{a}_\phi}{3.16} = 27.2\,\hat{a}_\phi$

$|\vec{E}_P| = 27.2\ V/m$

1) find $|\vec{E}|$ at $P(3,1,2)$ for the field of (4)

(a) two Co-axial Conducting cylinders $V = 50V$ at $\rho = 2m$ & $V = 20V$ at $\rho = 3m$.

$$\nabla^2 V = \frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) = 0$$

$\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) = 0$ Integ $\rho\frac{\delta V}{\delta\rho} = C_1 \Rightarrow \frac{\delta V}{\delta\rho} = \frac{C_1}{\rho}$.

Integ one more time $\Rightarrow V = C_1 \ln(\rho) + C_2$

$$\boxed{V = C_1 \ln(\rho) + C_2}$$

apply B.C

at $\rho = 2m$, $V = 50$

$\Rightarrow 50 = C_1 \ln(2) + C_2$ — (1)

at $\rho = 3m$, $V = 20$, $20 = C_1 \ln(3) + C_2$ — (2)

using eq (1) - eq (2)

$50 - 20 = C_1 \ln(2) + \cancel{C_2} - C_1 \ln(3) - \cancel{C_2}$

$30 = C_1 \ln(2) - C_1 \ln(3) = C_1 [\ln(2) - \ln(3)]$

$30 = C_1 \underbrace{\ln\left(\frac{2}{3}\right)}_{-0.405}$

$C_1 = \frac{30}{-0.405} = -74.07$

$V = C_1 \ln(\rho) + C_2$

$E = -\nabla V = -\frac{\delta V}{\delta\rho}\hat{a}_\rho = -\frac{\delta}{\delta\rho}\left[C_1 \ln(\rho)\right]\hat{a}_\rho$

$= -C_1 \hat{a}_\rho \left(\frac{1}{\rho}\right)$

$\rho = \sqrt{x^2+y^2} = \sqrt{(3)^2+(1)^2}$ $= \frac{-C_1\hat{a}_\rho}{\rho} = \frac{+74.07\hat{a}_\rho}{\rho}$

$= \sqrt{9+1} = \sqrt{10} = 3.16$

$\vec{E} = \frac{74.07\hat{a}_\rho}{3.16} = 23.4\hat{a}_\rho$

$|\vec{E}| = 23.4 \ V/m$

$$\nabla^2 V = \frac{-\rho_v}{\varepsilon}$$

$$\frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) = \frac{-\rho_v}{\varepsilon}$$

$$\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) = \frac{-\rho_v}{\varepsilon}\cdot\rho$$

Inte

$$\rho\frac{\delta V}{\delta\rho} = \frac{-\rho_v}{\varepsilon}\frac{\rho^2}{2} + c_1$$

$$\frac{\delta V}{\delta\rho} = \frac{-\rho_v}{2\varepsilon}\rho + \frac{c_1}{\rho}$$

$\therefore$ Inte

$$V = \frac{-\rho_v}{2\varepsilon}\frac{\rho^2}{2} + c_1\ln(\rho) + c_2$$

$$= \frac{-\rho_v}{4\varepsilon}\rho^2 + c_1\ln(\rho) + c_2$$

~~given the vector $\vec{E}$~~

$$\vec{E} = (12yx^2 - 6z^2x)\hat{a}_x + (4x^3 + 18zy^2)\hat{a}_y + (6y^3 - 6zx^2)\hat{a}_z$$

Check whether it represent a possible electric field

$$\nabla^2 V = \nabla\cdot\nabla V = \nabla\cdot(-\vec{E})$$

$$\nabla^2 V = -\nabla\cdot\vec{E}$$

$$= -\left(\frac{\delta}{\delta x}\hat{a}_x + \frac{\delta}{\delta y}\hat{a}_y + \frac{\delta}{\delta z}\hat{a}_z\right)\left[(12yx^2 - 6z^2x)\hat{a}_x + (4x^3 + 18zy^2)\hat{a}_y + (6y^3 - 6zx^2)\hat{a}_z\right]$$

$$= -\left[(24xy - 6z^2) + (0 + 36zy) + (0 - 6x^2)\right]$$

$$\nabla^2 V \neq 0$$

LE not satisfied thus there exists ρ_v as the region is not free of charge hence $\Rightarrow$ repre a possible electric field given $V = A \ln\left[\frac{B(1-\cos\theta)}{(1+\cos\theta)}\right] V$

S.T V satisfies L.Eq in Sphe coor.
find A & B S.T
$V = 100V$, $|\vec{E}| = 500 V/m$ at $r=5m$, $\theta = 90°$, $\phi = 60°$

Soln V-Satis

$$V = A \ln\left[\frac{B(1-\cos\theta)}{(1+\cos\theta)}\right] \quad (1)$$

V has only θ term

$$\nabla^2 V = \frac{1}{r^2 \sin\theta} \frac{\delta}{\delta\theta}\left(\sin\theta \frac{\delta V}{\delta\theta}\right) \Rightarrow \text{spherical co-ord.}$$

$$V = A \ln B + A \ln(1-\cos\theta) - A \ln(1+\cos\theta)$$

$$\therefore \frac{\delta V}{\delta\theta} = 0 + \frac{A\sin\theta}{1-\cos\theta} - A\frac{(-\sin\theta)}{1+\cos\theta} = A\sin\theta\left[\frac{2}{(1-\cos^2\theta)}\right] = \frac{2A\sin\theta}{\sin^2\theta}$$

$$\frac{\delta V}{\delta\theta} = \frac{2A}{\sin\theta}$$

$\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$

$1+\cos\theta - \cos\theta - \cos^2\theta$

$$\nabla^2 V = \frac{1}{r^2\sin\theta}\frac{\delta}{\delta\theta}\left[\cancel{\sin\theta}\left(\frac{2A}{\cancel{\sin\theta}}\right)\right] = \frac{1}{r^2\sin\theta} \times 0$$

satisfies.

$$\nabla^2 V = 0$$

to find A & B so that $V = 100V$, $|\vec{E}| = 500 V/m$ at $r = 5m$, $\theta = 90°$, $\phi = 60°$.

$\vec{E} = -\nabla V$

taking only θ term. in sphe co

$$\vec{E} = -\frac{1}{r}\left(\frac{\delta V}{\delta\theta}\right)\hat{a}_\theta$$

$$|\vec{E}| = -\frac{1}{r}\left(\frac{\delta V}{\delta\theta}\right)$$

already do

$$|\vec{E}| = -\frac{1}{r}\left(\frac{2A}{\sin\theta}\right)$$

$$500 = \left(\frac{-2A}{5\sin 90°}\right)$$

$$= \frac{-2A}{5}$$

$A = -1250$

$A = \frac{500 \times 5}{-2}$

from data $V = 100$, $\theta = 90°$ hence
Eq (1) becomes $\cos 90° = 0$

$100 = (-1250) \ln B$

$$\ln B = \left(\frac{-100}{1250}\right) = -0.08$$

natural logarithm

$B = \cancel{\frac{1}{\ln}(-0.08)}$ $B = e^{(-0.08)}$

= ans = 0.92

$\log B = ()$

$B = 10^{()}$

$$\nabla^2 V = \frac{-\rho_v}{\varepsilon}$$

$$\frac{1}{\rho}\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) = \frac{-\rho_v}{\varepsilon}$$

$$\frac{\delta}{\delta\rho}\left(\rho\frac{\delta V}{\delta\rho}\right) = \frac{-\rho_v}{\varepsilon}\cdot\rho$$

Inte

$$\rho\frac{\delta V}{\delta\rho} = \frac{-\rho_v}{\varepsilon}\frac{\rho^2}{2} + c_1$$

$$\frac{\delta V}{\delta\rho} = \frac{-\rho_v}{2\varepsilon}\rho + \frac{c_1}{\rho}$$

$\therefore$ Inte

$$V = \frac{-\rho_v}{2\varepsilon}\frac{\rho^2}{2} + c_1 \ln(\rho) + c_2$$

$$= \underline{\underline{\frac{-\rho_v}{4\varepsilon}\rho^2 + c_1\ln(\rho) + c_2}}$$

---

given the vector $\vec{E}$.

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4) two radial conducting planes $V = 50V$ at $\phi = 10^\circ$

$V = 20V$ at $\phi = 30^\circ$. $P(3,1,2)$ $|\vec{E}_P|$

$$\nabla^2 V = \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} = 0 \Rightarrow \frac{\partial^2 V}{\partial \phi^2} = 0$$

Integrate once $\frac{\partial V}{\partial \phi} = C_1$

Integ one more time $\boxed{V = C_1\phi + C_2}$

$$50 = C_1 10^\circ + C_2 \quad — (1)$$

$$20 = C_1 30^\circ + C_2 \quad — (2)$$

(1) − (2)

$$50 - 20 = C_1 10^\circ + \cancel{C_2} - C_1 30^\circ - \cancel{C_2}$$

$$30 = C_1 10^\circ - C_1 30^\circ$$

to convert into radian

$$30 = C_1[10^\circ - 30^\circ] \times \frac{\pi}{180^\circ}$$

$$30 = C_1[-20^\circ] \times \frac{\pi}{180}$$

$\frac{20}{180}$

$$C_1 = -\frac{30 \times 180}{\pi \times 20^\circ}$$

keep calci in radian mode. $\left(\frac{180^\circ}{20^\circ}\right) = 9$

$$C_1 = -\frac{30 \times 9}{\pi} = -85.987$$

$$\vec{E} = -\nabla V = \frac{-1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi = -\frac{1}{\rho}C_1\frac{\partial}{\partial \phi}(\phi)\cdot\hat{a}_\phi$$

$P(3,1,2)$

$\rho = \sqrt{x^2+y^2} = \sqrt{3^2+1^2} = \sqrt{10} = 3.16$

$$\vec{E} = \frac{-C_1\hat{a}_\phi}{\rho} = \frac{-(-85.987)\hat{a}_\phi}{}$$

Calci in radian mode

$$= \frac{85.987\hat{a}_\phi}{3.16} = 27.2\,\hat{a}_\phi$$

$$|\vec{E}_P| = 27.2\ V/m$$

1) find $|\vec{E}|$ at $P(3,1,2)$ for the field of (A)

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Integ one more time $\Rightarrow V = C_1 \ln(\rho) + C_2$

$$\boxed{V = C_1 \ln(\rho) + C_2}$$

apply B.C

at $\rho = 2m$, $V = 50$

$\Rightarrow 50 = C_1 \ln(2) + C_2$ — (1)

at $\rho = 3m$, $V = 20$, $20 = C_1 \ln(3) + C_2$ — (2)

using Eq (1) − Eq (2)

$50 - 20 = C_1 \ln(2) + \cancel{C_2} - C_1 \ln(3) - \cancel{C_2}$

$30 = C_1 \ln(2) - C_1 \ln(3) = C_1 [\ln(2) - \ln(3)]$

$30 = C_1 \ln\left(\frac{2}{3}\right)$ -0.405

$C_1 = \frac{30}{-0.405} = -74.07$

$V = C_1 \ln(\rho) + C_2$

$E = -\nabla V = -\frac{\delta v}{\delta\rho}\hat{a}_\rho = -\frac{\delta}{\delta\rho}\left[C_1 \ln(\rho)\right]\hat{a}_\rho$

$= -C_1 \hat{a}_\rho \left(\frac{1}{\rho}\right)$

$\rho = \sqrt{x^2+y^2} = \sqrt{(3)^2+(1)^2}$ $= \frac{-C_1 \hat{a}_\rho}{\rho} = \frac{+74.07\hat{a}_\rho}{\rho}$

$= \sqrt{9+1} = \sqrt{10} = 3.16$

$\vec{E} = \frac{74.07\hat{a}_\rho}{3.16} = 23.4\hat{a}_\rho$

$|\vec{E}_P| = 23.4 \ V/m$